

Bsp 16) a) $\frac{1}{2}$ b) 10 c) $\frac{a}{3}t^3 + \frac{b}{2}t^2 + ct$ d) 4 e) 0 f) 2
 g) $\frac{14}{3}$ h) 0 i) $\frac{q^3 - p^3}{3}$ j) $\frac{1}{a(a+1)}$ k) $\frac{3}{5} \cdot 0^{\frac{5}{3}}$ l) $\frac{e^2}{2} - \frac{1}{2e^2}$

Bsp 17) a) $-\frac{1}{5}x^{-5} + C$ b) $\frac{2}{3}t^{\frac{2}{3}} + C$ c) $\frac{u^2}{2} + \frac{2u^3}{3} + \frac{3u^4}{4} + C$

d) $\frac{1}{1,1} \cdot s^{1,1} + C$ e) $-A \cdot \cos \theta + B \cdot \sin \theta + C$ f) $-\frac{1}{12Q}$

g) $p \cdot \frac{t^2}{2} + q \cdot \ln t + C$

Bsp 18) $\frac{k}{2} \cdot \sigma^2$

Bsp 19) a) $y'' = -6x$ b) $u'' = 40z^3 - 18z$ c) $W'' = 6t^{-3}$ d) $p'' = -\frac{1}{2} \cdot \sigma^{-\frac{3}{2}}$

Bsp 20) a) $6ax$ b) $-\frac{2}{9} \cdot t^{-\frac{4}{3}}$ c) $-\frac{1}{4}(u+2)^{-\frac{3}{2}}$ d) $4 \cos(2\alpha+3)$
 e) $-\cos \theta + \sin \theta$ f) $-\frac{2}{p}(r-1)^{-\frac{5}{3}}$ g) $P e^{3x}$ h) $-c^2(ct+1)^{-2}$ i) $e^{\sin x} \cdot \cos x$

Bsp 21) a) $y'' = 1 \Rightarrow$ konvex nach oben b) $F'' = -2 \Rightarrow$ konvex nach unten

c) $v'' = -\frac{2}{3} \Rightarrow$ konvex nach unten d) $y'' = 0 \Rightarrow$ linear

e) $y'' = 6x \Rightarrow x > 0$: konvex nach oben
 $x < 0$: konvex nach unten

Bsp 22) $f(x) = ax^2 + bx + c$
 $f'(x) = 2ax + b$... linear
 $f''(x) = 2a$... konstant

Bsp 23) $s(t) = a \cdot \frac{t^2}{2}$
 $v(t) = a \cdot t$

Bsp 24) a) $x = \pm \frac{1}{\sqrt{3}}$ b) $t = \sqrt[3]{\frac{1}{2}}$ c) $p = \frac{1}{2}$

Bsp 25) a) $t_1 = -1$: Maximum $t_2 = 1$: Minimum
 b) $s_1 = 0$: Minimum $s_2 = \frac{2}{3}$: Sattelpunkt
 c) $\alpha_1 = \frac{\pi}{4}$: Maximum $\alpha_2 = \frac{3\pi}{4}$: Minimum

Bsp 26) a) $x_1 = \frac{3}{2}$ abs. Extremum
 $x_2 = 0$ lokales Ext.
 $x_3 = 5$ lokales Ext.
 b) $x_1 = 3$ abs. Extremum
 $x_2 = -5$ abs. Ext.
 $x_3 = 0$ lokales Ext.
 $x_4 = \infty$ lokales Ext.

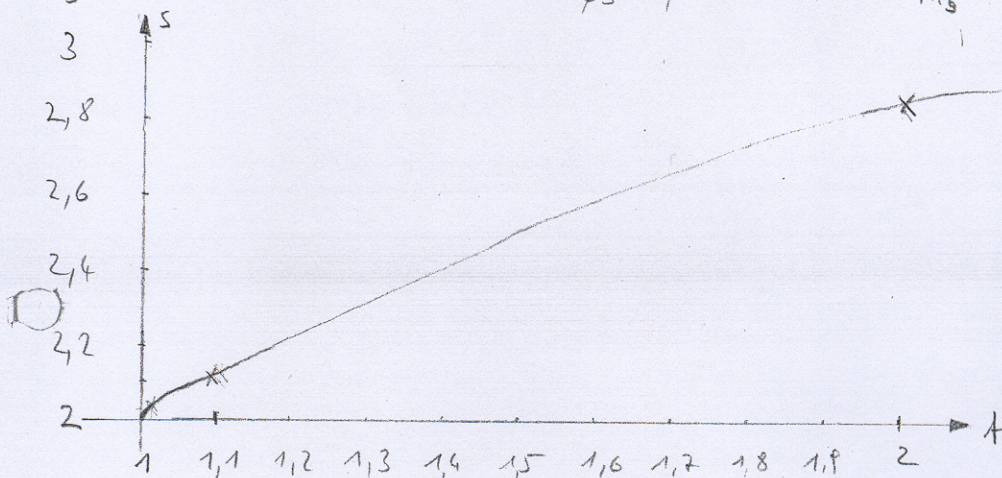
Bsp 27) $x_{\max} = 5 \text{ cm}$; $M = 166,6 \text{ mg}$ Bsp 28) $\text{H}_3\text{O}^+_{\min} = 10^{-7} \text{ mol}$

Bsp 29) $x = \frac{a}{2}$ Bsp 30) $v = \sqrt{\frac{W^2}{3 \rho^2 S A}}$

Lösungen (Kapitel 2): (ohne Gewähr!)

Bsp 1)

$t_1 = 1s:$	$s_{m1} = 2m$	$v_{\phi 1} = 2m/s$	$v_{m1} = 2m/s$
$t_2 = 1,001s:$	$s_{m2} = 2,001m$	$v_{\phi 2} = 0,99975m/s$	$v_{m2} = 1,9995m/s$
$t_3 = 1,01s:$	$s_{m3} = 2,0098m$	$v_{\phi 3} = 0,997512m/s$	$v_{m3} = 1,99007m/s$
$t_4 = 1,1s:$	$s_{m4} = 2,09762m$	$v_{\phi 4} = 0,976177m/s$	$v_{m4} = 1,90693m/s$
$t_5 = 2s:$	$s_{m5} = 2,82843m$	$v_{\phi 5} = 0,828427m/s$	$v_{m5} = 1,41421m/s$



Bsp 2) a) $y' = 2x$ b) $y' = 3x^2$ c) $y' = 21x^6 - 4x^3 + 5$

Bsp 3) a) $y' = -2x^{-3}$ b) $u' = 5w^4$ c) $v' = \frac{2}{3}z^{-\frac{1}{3}}$ d) $m' = -3t^{-4}$

e) $\Delta' = \frac{4}{3}Q^{-\frac{2}{3}}$ f) $z' = -\frac{1}{2}p^{-\frac{3}{2}}$ g) $y' = 2$ h) $y' = 2x^{\frac{5}{3}}$

i) $y' = 2 \cdot \cos(2x)$ j) $y' = -2\cos x \cdot \sin x$ k) $y' = \frac{\sin x}{\cos^2 x}$ l) $y' = 2x \cdot \tan x + x^2 \cdot \frac{1}{\cos^2 x}$

m) $y' = -3\sin x - 2\cos x$ n) $y' = 2 \cdot \sin(x + \frac{\pi}{2}) \cdot \cos(x + \frac{\pi}{2})$ o) $y' = -\cot^2 x - 1$

p) $Q'(t) = -a \cdot \sin t + \frac{1}{t^2}$ q) $P'(s) = -1 - \frac{1}{2} \cdot \cos s$ r) $\lambda'(x) = \sin x$

Bsp 4)

a) $y' = 2x + 4$

b) $y' = \sin x + x \cdot \cos x$

c) $z'(t) = -\cos t - (1+t) \sin t$

d) $Q'(x) = \cos^2 x - \sin^2 x$

e) $n'(x) = \frac{1}{3}x^{-\frac{2}{3}}(1-2x) - 2x^{\frac{1}{3}}$

f) $f'(y) = \frac{8}{2} \cdot y^{\frac{1}{2}} \cdot \sin y + 8 \cdot y^{\frac{1}{2}} \cdot \cos y$

Bsp 5)

$y' = 3x^2 - 4x + 1$

Bsp 6) $y' = 8x + 12$

Bsp 7)

a) $y' = 2(x+5)$

b) $y' = 2(u^2-3) \cdot 2u$

c) $s' = -(t-2)^{-2}$

d) $K' = 2(1-v)^{-2}$

e) $v' = -\frac{3}{2}(4-3t)^{-\frac{1}{2}}$

f) $p' = 4 \cos(4x-5)$

Bsp 8)

a) $y' = 2 \cos(2x-4)$

b) $u'(\beta) = -3p \cdot \sin(3\beta)$

c) $R'(s) = -a(a s + b)^{-2}$

d) $z' = \frac{3}{2}(3x+1)^{-\frac{1}{2}}$

e) $u'(x) = -4(x-1)^{-5}$

f) $E'(w) = -\frac{1}{w^2} - \frac{1}{(w-1)^2}$

g) $z' = \frac{1}{2}(t-1)^{-\frac{1}{2}} - \frac{1}{2}(t+1)^{-\frac{3}{2}}$

h) $f'(x) = 2x \cdot \cos x (\cos x - x \cdot \sin x)$

Bsp 9) a) $y' = \frac{3x^2(1-x) + x^3}{(1-x)^2}$ b) $s'(t) = \frac{a(ct+d) - c(at+b)}{(ct+d)^2}$ c) $Q'(x) = \frac{\sin x - \cos x - 1}{(1 + \cos x)^2}$

d) $T'(t) = \frac{4 + 3t^2}{(1+t^2)^2}$ e) $f'(u) = (pu^3 + q)^{-\frac{2}{3}} \cdot pu^2$ f) $h'(p) = \frac{2\cos(2p) \cdot \cos(3p) + 3\sin(2p) \cdot \sin(3p)}{(\cos(3p))^2}$

Bsp 10) a) $y'(3) = \frac{1}{\ln 10}$ b) $y'(3) = -2$ c) $y'(1) = \frac{5}{2}$ d) $y'(10) = \frac{1}{20 \cdot \ln 10}$

Bsp 11) a) $y'(1) = 10 + 10 \cdot \ln 10$ b) $y'(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} \cdot e^{\frac{1}{\sqrt{2}}}$ c) $y'(1) = \frac{e}{2}$

d) $y'(1) = 1$ e) $y'(2) = 64 \cdot \ln 2$ f) $y'(1) = 3^{\lg 1} \cdot \frac{\ln 3}{\ln 10}$

Bsp 12) a) $2\sqrt{at} + (2t-3) \cdot \frac{a}{2} \cdot \frac{1}{\sqrt{at}}$ b) $\frac{1-u}{(1-u)^2}$ c) $-\frac{z}{1-z}$

d) $3(\lambda \sin(a\lambda))^2 \cdot (\sin(a\lambda) + \lambda a \cos(a\lambda))$ e) $3e^{3x}$ f) $-2e^{1-2u}$

g) $\frac{5}{5x+4}$

h) $\frac{1}{\lambda(1-\lambda)}$

i) $\ln(3u) + 1$

j) $3e^{3x}$

k) $e^{\frac{4}{x}} \cdot (1 + \frac{4}{x^2})$

Bsp 13) (- Beweis -)

Bsp 14) a) $y = 2x^3 + C$ b) $y = 4x^2 - 7x + C$ c) $u = \frac{a}{2}t^2 + bt + C$

d) $y = \frac{5}{4}x^4 + C$ e) $w = t^2 - 8t + C$ f) $u = u_0 x + \sin x + C$

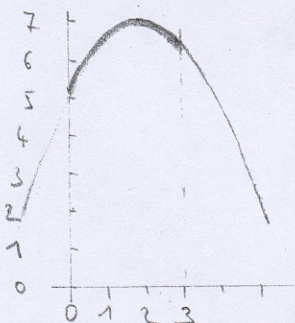
g) $y = \frac{1}{3}\sin t + C$ h) $u = \frac{1}{2} \cdot \sin 2x + C$ i) $K = -\frac{1}{u} + C$

j) $y = \frac{3}{5}x^5 - \frac{1}{3}x^3 + C$ k) $y = x - \frac{3}{5}x^{\frac{5}{3}} + C$ l) $y = \frac{a}{4}x^4 - \frac{b}{6}x^6 + C$

m) $s = \frac{2a}{3} \cdot t^{\frac{3}{2}} + C$ n) $u = -\frac{k}{2s^2} + C$ o) $Q(x) = \frac{a^2}{2} \cdot \cos x + ab \cdot \sin x + C$

p) $y = pt - \frac{g}{4} + C$ q) $P = \frac{a}{3}v^3 + \frac{b}{2}v^2 + cv - \frac{d}{v} + C$

Bsp 15) $F = 19,5$



Bsp 31) $r = \sqrt[3]{\frac{V}{2\pi}}$; $h = \sqrt[3]{\frac{4V}{\pi}}$; $r:h = 1:2$

Bsp 32) $h_{\max} = 0,99 \text{ m}$; $v(0) = 4,4 \text{ m/s}$; $a = -9,8 \text{ m/s}^2$

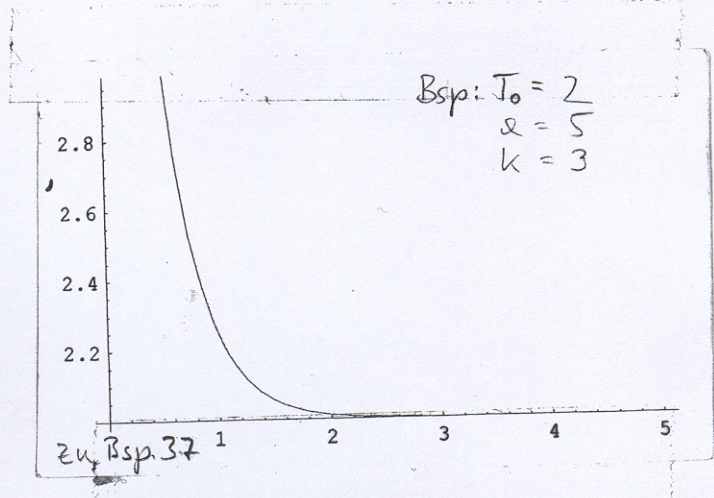
Bsp 33) $V(r) : V(\delta r) = 1 : \delta^3$
 $O(r) : O(\delta r) = 1 : \delta^2$

Bsp 34) a) $8040,4$ b) 0 c) $1,464$
d) $-2\sqrt{3-u} + C$ e) $\frac{3}{4} \cdot \frac{1}{b} \cdot (a+bv)^{\frac{4}{3}} + C$ f) $\frac{1}{2} \cdot \left(\frac{x}{3} - 1\right)^6 + C$

Bsp 35) a) $-x \cdot \cos x + \sin x + C$ b) $-0,7879$ c) $\frac{x^3}{3} \left(\ln x - \frac{1}{3}\right) + C$

Bsp 36) a) $e^{x \cdot \ln 2}$ b) $e^{-0,431x}$ c) $e^{-3,08894}$

Bsp 37) $\frac{dT}{dt} = -\alpha k \cdot e^{-kt}$



Bsp 39) a) $y = \frac{2e^{3t} \cdot C - 5}{e^{3t} \cdot C - 1}$
b) $u = \frac{3}{1 - e^{\frac{3}{2} \cdot t} \cdot C}$

Bsp 40) a) $y = \frac{3 + 3e^{6t} \cdot C}{1 - e^{6t} \cdot C}$ b) $y = \frac{4 + 3e^{7x} \cdot C}{1 - e^{7x} \cdot C}$

Bsp 42) (-Beweis-) Bsp 43) (-Beweis-)

Bsp 44) $0,7958 \text{ l}$

Bsp 45) $f''(x) = \frac{1}{12\pi} \cdot e^{-\frac{x^2}{2}} \cdot (x^2 - 1)$; $x_{1/2} = \pm 1$

Bsp 46) $2,7646 \text{ ml/s}$

Bsp 47) $\frac{p_0 \cdot q}{\beta} \cdot (1 - e^{-\beta \cdot h})$

Bsp 48) $\frac{3A_0}{\alpha} \cdot (e^{21\alpha} - 1) \text{ cm}^2$

Bsp 49) $\beta = 0,0000843551$ $V = 343784,811 \text{ l}$