
Special Classes of Copulas and their Properties

Abstract

This dissertation contains four articles (three of which are already published) in which we dealt with some interesting questions on copulas. In the first article we defined a quantification of asymmetry with respect to the metric D_1 as introduced by Trutschnig (2011) and studied analogous questions as established by Nelsen (2007) and Klement and Mesiar (2006), who worked with the standard uniform metric d_∞ . In particular, we gave a characterization of all copulas C maximizing $D_1(C, C^t)$, i.e., copulas C for which C has maximal distance to the transpose C^t of C . Despite the difficult calculation of the metric D_1 , we also provided a handy representation of all copulas C maximizing $D_1(C, C^t)$. This representation can be used to show the existence of copulas with full support maximizing $D_1(C, C^t)$, and to provide a comparison of D_1 - and d_∞ -asymmetry. In the second article we studied the class of so-called reflected maxmin (RMM) copulas as recently introduced by Košir and Omšadič (2019), which can be regarded as simplest possible representation of so-called maxmin copulas. Using Markov kernels we showed that the class of RMM copulas is compact in (\mathcal{C}, d_∞) , we characterized absolutely continuous RMM copulas, and showed that the limit of a sequence of absolutely continuous RMM copulas is absolutely continuous too. Furthermore, we determined formulas for Spearman's rho and Kendall's tau of RMM copulas and provided some new inequalities for rho and tau. In the third article we solved one of two open questions posed by Prof. Christian Genest at the Salzburg workshop on Dependence Models and Copulas in 2016 concerning the bounds on Pickands dependence functions of extreme-value copulas with a given value of Spearman's rho or Kendall's tau. It is well-known that a bivariate extreme-value copula can be characterized by a convex function of one variable which is called a Pickands dependence function. We derived the smallest compact set containing the graphs of all Pickands dependence functions with a fixed value of Spearman's rho or Kendall's tau. Finally, in the fourth manuscript we restricted the class of all extreme-value copulas to the class of all symmetric extreme-value copulas and studied analogous questions as in the third article. Moreover, we provided a relation among copula-based dependence measures such as Spearman's rho, Kendall's tau, and Joe's coefficient of upper-tail dependence for (symmetric) extreme-value copulas.