

Gastvortrag

Montag, 15. Mai 2017
13.15 Uhr
Seminarraum II

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Harmonical data from a mathematical perspective

Abstract:

\mathbb{R} -moulds of numerical semigroups are defined as increasing sequences of real numbers whose discretizations may give numerical semigroups. The ideal sequence of musical harmonics is an \mathbb{R} -mould and discretizing it is equivalent to defining equal temperaments. The number of equal parts of the octave in an equal temperament corresponds to the multiplicity of the related numerical semigroup.

Analyzing the sequence of musical harmonics two important properties of moulds are derived, those of being metric and fractal. It is demonstrated that, up to normalization, there is only one metric mould and one non-bisectional fractal mould. Furthermore, it is shown that the unique non-bisectional fractal mould is given by the golden ratio.

The case of half-closed cylindrical pipes imposes to the sequence of musical harmonics one third property, the so-called even-filterability property.

We will prove that the maximum number of equal divisions of the octave such that the discretizations of the metric mould and the non-bisectional fractal mould coincide, and such that the discretization is half-closed-pipe admissible is 12. This result gives a mathematical justification for the choice of 12 as the maximum number of equal divisions of the octave in equal temperaments.