UNIVERSITÄT SALZBURG

Fachbereich Mathematik
Hellbrunnerstraße 34
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Gastvortrag

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15.15 Uhr
Seminarraum II

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An adaptive hp-XFEM method for a Hardy problem featuring an inverse square point potential

Abstract:
We consider a partial differential equation with the operator $-\Delta - \frac{\lambda}{|x|^2}$ in two dimensions, which depends on a parameter λ. To answer the question of existence and uniqueness in the presence of the inverse square potential, we use HARDY inequalities [1] and hereby extend the existing theory in classic SOBOLEV-spaces $H_0^1(\Omega)$ to spaces $H_{\Gamma_0}^1(\Omega)$, allowing for non-zeros traces. In particular we introduce geometrical scenarios, in which the best HARDY constant can be recovered. The inverse square potential cannot be considered as a lower order perturbation of the LAPLACE operator, hence yielding to several problems from both the theoretical and numerical point of view. Based on the analysis in Kondrat'ev spaces we are able to give an explicit decomposition of the solution in locally conical domains. As a conclusion the inverse square operator can cause arbitrary worse singular solutions in the sense of $H^{1+\epsilon}(\Omega)$-regularity, where $\epsilon \to 0$, when λ tends to the HARDY constant.

Although it is clear, that standard uniform FEM approaches are not suitable, the use of the *hp*-adaptive finite element method also yields to multiple difficulties related to approximation quality and the coercivity constant. To recover good approximation properties we apply an eXtended *hp*-adaptive FEM that is enriching the *hp*-adaptive FEM space with the first singularities. This strategy allows for exponential convergence rates bounded away from zero independent of λ. The introduced problem of numerical quadrature in the XFEM framework is then solved with the help of a generalized Duffy transformation [2]. Moreover, we discuss the challenges for a-posteriori error estimation with the inverse square potential.

[1] G. H. Hardy, J. E. Littlewood, and G. Pólya. *Inequalities*. Cambridge university press, 1952.
[2] S. E. Mousavi and N. Sukumar. *Generalized Duffy transformation for integrating vertex singularities*. *Computational Mechanics*, 45(2-3):127–140, 2010.

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