UNIVERSITÄT SALZBURG

Fachbereich Mathematik
Hellbrunnerstraße 34
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Gastvortrag

Freitag, 9. Oktober 2015
14.15 Uhr
Seminarraum II

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Composite finite elements for transport dominated flow problems

Abstract:

Composite finite elements are based on star-shaped polyhedral mesh cells, i.e., a mesh cell can be, for instance, a quadrilateral, a hexahedron or a prismatic cell. A basic feature of composite finite elements is that each polyhedral mesh cell is decomposed by means of simplices using the a star-shape center point which is connected to the boundary vertices of the polyhedral cell. It is essential that the arising global simplicial submesh is regular without any hanging nodes.
In the assembling procedure for each composite element, there will be executed an internal loop over all simplicial subcells and the interior degrees of freedom (dof's) are removed out of the global system of equations by means of static condensation. Thus, the number of global unknowns, the required memory for the global sparse matrix and the time for the solution of the global linear system are reduced essentially.
In the talk, we discuss two model problems. The first one is the scalar pure transport problem and the second one the viscosity dependent 'Stokes-Brinkman' problem. The combination of both techniques will then lead to a stabilized and accurate discretization method for the Oseen model problem representing a linearization of the time-dependent Navier-Stokes equations.
As a stabilized discretization we apply a so-called "local Continuous Interior Penalty" (local CIP) method. Here, to the standard Galerkin discretization terms there are added for each simplicial face, that is located in the interior of the composite cell, face integrals over the gradient jumps of the trial and test functions. The big advantage of this local CIP over the original CIP is that it allows for static condensation in contrast to the classical CIP method. If the degrees of freedom in the interior of the composite finite elements are eliminated using static condensation then the resulting couplings of the skeleton degrees of freedom are comparable to those for classical conforming finite element methods which leads to a substantially smaller matrix stencil than for the standard global CIP-method. Optimal stability and error estimates can be proved also for the local CIP version. In the talk, the results of numerical tests for the new method are presented.
For the Stokes-Brinkman model, our error bound for the local CIP method does not increase if the viscosity parameter tends to zero which is mainly caused by adding a penalty term for the divergence of the velocity in the discretization. Moreover, the reduction effect of the static condensation is much stronger for this model since, beside of the elimination of all velocity degrees of freedom in the interior of each composite cell, all pressure degrees of freedom except for the cell-wise constants can be eliminated. Also for this model problem we present the results of numerical tests.

Einladender: Andreas Schröder