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Measuring international uncertainty using global vector autoregressions with drifting parameters

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Measuring international uncertainty using global vector autoregressions with drifting parameters

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This paper investigates the time-varying impacts of international macroeconomic uncertainty shocks. We use a global vector autoregressive (GVAR) specification with drifting coefficients and factor stochastic volatility in the errors to model six economies jointly. The measure of uncertainty is constructed endogenously by estimating a scalar driving the innovation variances of the latent factors, and is included also in the mean of the process. To achieve regularization, we use Bayesian techniques for estimation, and introduce a set of hierarchical global-local shrinkage priors. The adopted priors center the model on a constant parameter specification with homoscedastic errors, but allow for time-variation if suggested by likelihood information. Moreover, we assume coefficients across economies to be similar, but provide sufficient flexibility via the hierarchical prior for country-specific idiosyncrasies. The results point towards pronounced real and financial effects of uncertainty shocks in all countries, with differences across economies and over time.

JEL: C11, C55, E32, E66, G15
KEYWORDS: Bayesian global vector autoregressive model, state space modeling, hierarchical priors, factor stochastic volatility, stochastic volatility in mean

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1. INTRODUCTION

Uncertainty has received a substantial amount of attention as a driving force of business cycle fluctuations following the experiences of economists and policy makers in the aftermath of the Great Recession. Measuring uncertainty and its impact on the economy is the subject of numerous articles, with prominent contributions including Fernández-Villaverde *et al.* (2015), Jurado *et al.* (2015), Ludvigson *et al.* (2015), Caldara *et al.* (2016), Baker *et al.* (2016), Basu and Bundick (2017), Fajgelbaum *et al.* (2017), Schaal (2017), Bloom *et al.* (2018), and Carriero *et al.* (2018b), among others.[1] These studies provide compelling theoretical and empirical evidence suggesting negative economic consequences of uncertainty shocks. Elevated levels of uncertainty can produce large drops in economic activity, and moreover render counteracting monetary and fiscal policies less effective (see, for instance, Aastveit *et al.*, 2013; Bertolotti and Marcellino, 2019). Transmission channels of uncertainty shocks to the macroeconomy relate mainly to real phenomena in the traditional literature, such as distorted corporate decision making (Bernanke, 1983; Bloom, 2009), while recent papers highlight the importance of disturbances on credit and financial markets (Gilchrist and Zakrajšek, 2012; Gilchrist *et al.*, 2014; Alessandri and Mumtaz, 2019).

The measurement of uncertainty is a non-trivial task, stemming from its fundamentally unobservable nature. Many researchers construct proxies for uncertainty (e.g. stock market volatilities, or the occurance of uncertainty related keywords in newspapers), and treat them as observed in subsequent analyses. Approaches relying on such measures are critizised by Carriero *et al.* (2018b) for several reasons, with incorrect statistical inference in two-step econometric frameworks, and measurement errors biasing the results among them (see also Carriero *et al.*, 2015a;b). Methods proposed to alleviate these concerns include variants of stochastic volatility in mean (SVM) models. This modeling approach assumes time variation in the second moments of shocks to economic series, that also affect the respective first moments in dynamic time series models (see Koopman and Hol Uspensky, 2002). The time-varying volatilities are considered as a measure of uncertainty, establishing a unified framework for estimating uncertainty and its effects jointly. Econometric studies featuring variants of this approach are, for instance, Berument *et al.* (2009), Mumtaz and Zanetti (2013), Carriero *et al.* (2018b), Mumtaz and Surico (2018), or Alessandri and Mumtaz (2019).

Even though the literature on the impact of uncertainty shocks appears voluminous, most of the relevant contributions are confined to single-country analysis, and assume model parameters other than the time-varying volatilities to be constant over time.[2] Both of these limitations in general may be considered overly restrictive: A growing number of papers suggests the presence of structural breaks in many economic time series, a feature that requires flexible econometric specifications to obtain reliable inference. Popular methods to deal with such dynamics are time-varying parameter models that allow for drifting coefficients in addition to stochastic volatilities (see, for instance, Cogley and Sargent, 2005; Primiceri, 2005; D'Agostino *et al.*, 2013; Koop and Korobilis, 2013; Aastveit *et al.*, 2017; Chan and Eisenstat, 2018; Huber *et al.*, 2019).

Besides structural breaks in model parameters, there exists substantial evidence on the importance of taking global linkages, spillovers, and feedback effects between economies into account. Neglecting cross-border relationships entails the risk of omitted variable bias and may obscure important transmis-

[1] A comprehensive survey of the related literature is provided by Bloom (2014).

[2] For notable exceptions in terms of nonlinear modeling, see Mumtaz and Theodoridis (2018) and Alessandri and Mumtaz (2019). Examples for multi-economy modeling frameworks include Mumtaz and Theodoridis (2015), Crespo Cuaresma *et al.* (2017), Mumtaz and Theodoridis (2017), Rossi and Sekhposyan (2017), and Carriero *et al.* (2018a).

sion channels of shocks. Multi-economy frameworks proposed to study international macroeconomic dynamics include factor models (see Kose *et al.*, 2003; Mumtaz and Surico, 2009), panel VARs (PVARs, see Canova and Ciccarelli, 2004; 2009; Koop and Korobilis, 2016) and global VARs (GVARs, see Pesaran *et al.*, 2004; Dees *et al.*, 2007; Eickmeier and Ng, 2015; Feldkircher and Huber, 2016; Huber, 2016).

Motivated by the notions above, this paper proposes a multi-economy model with drifting coefficients and factor SVM to estimate uncertainty and its effects on a set of economies jointly. The contributions of this article are both of empirical and methodological nature. From an empirical perspective, we estimate an international measure of uncertainty and use the endogenous volatility-based measure to simulate dynamic responses for multiple economies and variable types to an international uncertainty shock. Similar to Carriero *et al.* (2018b) for the United States, the employed specification discriminates between uncertainty common to a large set of macroeconomic and financial indicators, while also featuring series-specific idiosyncrasies. The paper is also similar to Crespo Cuaresma *et al.* (2017), who rely on a factor stochastic volatility specification to measure uncertainty and assess the international effects of uncertainty shocks. By contrast, using a time-varying parameter multi-country VAR allows for studying whether the implications of volatility shocks changes over time.

From an econometric perspective, the paper provides several modeling contributions. First, we extend the GVAR model of Pesaran *et al.* (2004) to account for time-varying static and dynamic interdependencies between economies (for a similar approach, see Crespo Cuaresma *et al.*, 2019). The GVAR specification serves as a parsimonious framework to impose sensible parametric restrictions in large-scale multi-country models. Second, for capturing international financial sectors, we augment the basic setup with a term-structure model for interest rates in the spirit of Nelson and Siegel (1987). Though this modeling framework decreases the number of parameters compared to unrestricted estimation substantially, the parameter space of the model is still high-dimensional. As a remedy and third contribution, we employ Bayesian methods and adapt global-local priors designed for achieving shrinkage in time-varying parameter models (see Frühwirth-Schnatter and Wagner, 2010; Belmonte *et al.*, 2014; Bitto and Frühwirth-Schnatter, 2019). Finally, for measuring uncertainty endogenously, we follow Crespo Cuaresma *et al.* (2017) and model the high-dimensional variance covariance matrix of the system using a factor stochastic volatility structure. The proposed measure of uncertainty is a scalar driving the variance of the common factors. The model can thus be considered a multivariate extension of the SVM model with time-varying parameters by Chan (2017).

Bayesian inference is obtained by constructing a hierarchical prior that efficiently exploits cross-sectional information. In particular, the country-specific coefficients are assumed to arise from a common distribution, capturing that domestic dynamics across countries are similar. This approach provides a link to the literature on the Bayesian treatment of panel data, related to the random coefficients and heterogeneity model (Verbeke and Lesaffre, 1996; Allenby *et al.*, 1998; Frühwirth-Schnatter *et al.*, 2004). Moreover, we impose a global-local shrinkage prior on the common mean, allowing to push less important coefficients towards zero. Combined with the non-centered parameterization for state space models set forth in Frühwirth-Schnatter and Wagner (2010), this setup allows to test a set of parametric restrictions. First, we stochastically select which coefficients are non-zero. Second, we identify which coefficients can be set to zero in a data driven fashion, and which of them are heterogeneous and homogeneous across countries. Third, the prior shrinks the model towards a constant parameter specification when suggested by likelihood information. Imposing a similar shrinkage prior also on the innovation variances of the stochastic volatility state equations allows to center the system on homoscedastic errors. Flexible local scaling parameters preserve the possibility of heteroscedasticity across idiosyncratic series, if required.

Our model is applied to monthly data for six economies (France, Germany, United Kingdom, Italy, Japan, and the United States) for the period ranging from 1991:04 to 2018:07. The information set includes several recessionary episodes, and thus periods of economic distress when uncertainty is typically perceived to play a major role.[3] The endogenous measure of uncertainty is comparable to established proxies, and links well to events associated with high uncertainty. Besides macroeconomic uncertainty that is common to all series across all considered economies, we find various interesting patterns and idiosyncratic events in variable-specific volatilities.

Impulse responses shed light on the consequences of uncertainty shocks to a set of macroeconomic and financial quantities. Here, one key insight is that the responses for prices, unemployment, industrial production and equity prices are heterogeneous across the six countries in terms of magnitude and timing. In general, we find that uncertainty shocks exert disinflationary pressure, increase unemployment, depress industrial production and negatively affect equity prices, in line with the established literature. We provide evidence for time-varying consequences of uncertainty shocks. Some variables show systematic declines in their responsiveness to uncertainty shocks while the responses remain comparatively stable over time for others, corroborating findings in Mumtaz and Theodoridis (2018). For selected quantities in a subset of countries, the time-varying effects of uncertainty shocks do not evolve gradually, but exhibit distinct features for specific periods, as discussed for instance in Alessandri and Mumtaz (2019).

The article is structured as follows. Section 2 proposes the global vector autoregressive model with drifting coefficients and factor SVM to analyze the impact of uncertainty shocks across multiple economies. This section includes details on the Bayesian econometric framework. Section 3 presents the data and discusses model specification. Section 4 investigates the uncertainty measure and provides a discussion of the empirical results. Section 5 concludes.

2. ECONOMETRIC FRAMEWORK

In this section, we set forth a parsimonious multi-country model to measure the international consequences of uncertainty shocks on macroeconomic and financial variables for a set of economies. We first discuss the general setup and proceed with the specification for the drifting coefficients and time-varying volatilities. The section also contains information on the prior setup and the sampling algorithm.

2.1. *Model specification*

Let $\boldsymbol{y}_{it}$ denote a $k \times 1$ vector of endogenous variables for $t = 1, \ldots, T$ specific to country $i = 1, \ldots, N$. Collecting country-specific endogenous variables yields the $K \times 1$ vector $\boldsymbol{y}_t = (\boldsymbol{y}'_{1t}, \ldots, \boldsymbol{y}'_{Nt})$ with $K = kN$, while we stack the reduced form shocks to $\boldsymbol{y}_{it}$ in a $K \times 1$ vector $\boldsymbol{\epsilon}_t = (\boldsymbol{\epsilon}'_{1t}, \ldots, \boldsymbol{\epsilon}'_{Nt})'$. Following Aguilar and West (2000) and Kastner and Huber (2018), we consider a factor stochastic volatility structure on the error term,

$$\boldsymbol{\epsilon}_t = \boldsymbol{L}\boldsymbol{f}_t + \boldsymbol{\eta}_t, \quad \boldsymbol{f}_t \sim \mathcal{N}(\boldsymbol{0}, \exp(h_t) \times \boldsymbol{\Sigma}), \quad \boldsymbol{\eta}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Omega}_t). \tag{1}$$

[3] In particular, relevant events are the 1997 Asian financial crisis, the early 2000s recession related to the burst of the Dot-com bubble, the global financial crisis, the Great Recession, and the subsequent European sovereign debt crisis.

Here, $\boldsymbol{f}_t$ is a vector of $d \times 1$ common static factors (with $d \ll K$), and $\boldsymbol{\eta}_t$ an idiosyncratic white noise shock vector of dimension $K \times 1$. Latent factors are linked to the errors by the $K \times d$ factor loadings matrix $\boldsymbol{L}$. The factors $\boldsymbol{f}_t$ are Gaussian with zero mean and common time-varying volatility $\exp(h_t)$ scaling a diagonal $d \times d$ matrix $\boldsymbol{\Sigma} = \boldsymbol{I}_d$, with $\boldsymbol{I}_d$ referring to a d-dimensional identity matrix.

The idiosyncratic error components $\boldsymbol{\eta}_t$ are assumed to follow a Gaussian distribution centered on zero with a $K \times K$ time-varying diagonal variance covariance matrix $\boldsymbol{\Omega}_t = \text{diag}(\exp(\omega_{1t}), \dots, \exp(\omega_{Kt}))$. Note that the sign and scale of the factors and their loadings are not econometrically identified. We achieve identification by setting the upper $d \times d$ block of $\boldsymbol{L}$ to a lower triangular matrix with ones on the diagonal.

For both the volatility of the factors and the variances of the idiosyncratic component of the decomposed error term, we rely on a stochastic volatility model (see, for instance, Jacquier *et al.*, 2002). Here, h_t and $\omega_{ij,t}$ for $i = 1, \dots, N$ and $j = 1, \dots, k$ follow independent autoregressive processes. As in Primiceri (2005), we assume a random walk specification

$$h_t = h_{t-1} + \xi_t, \quad \xi_t \sim \mathcal{N}(0, \sigma_h) \tag{2}$$

$$\omega_{ij,t} = \omega_{ij,t-1} + \zeta_t, \quad \zeta_t \sim \mathcal{N}(0, \sigma_{\omega ij}) \tag{3}$$

with σ_h and $\sigma_{\omega ij}$ denoting the state-equation innovation variances.[4] Note that for the case of σ_h and $\sigma_{\omega ij}$ equal to zero, we obtain homoscedastic errors. We exploit this notion below by rewriting the model in its non-centered parameterization (see Frühwirth-Schnatter and Wagner, 2010). This allows us to impose flexible shrinkage priors for stochastically selecting whether time-varying volatilities are required to adequately fit the data.

The dynamic evolution of $\boldsymbol{y}_{it}$ is governed by a vector autoregressive (VAR) process with drifting coefficients and features the common volatility of the factors in the mean:

$$\boldsymbol{y}_{it} = \boldsymbol{\alpha}_{it} + \sum_{p=1}^{P} \boldsymbol{A}_{ip,t} \boldsymbol{y}_{it-p} + \sum_{q=1}^{Q} \boldsymbol{B}_{iq,t} \boldsymbol{y}^*_{it-q} + \boldsymbol{\beta}_{it} h_t + \boldsymbol{\epsilon}_{it}. \tag{4}$$

Here, we define the $k \times 1$ intercept vector $\boldsymbol{\alpha}_{it}$ and $k \times k$ coefficient matrices $\boldsymbol{A}_{ip,t}$ $(p = 1, \dots, P)$. To establish dynamic interdependencies between economies in the spirit of the GVAR model (Pesaran *et al.*, 2004), we construct a $k \times 1$-vector $\boldsymbol{y}^*_{it} = \sum_{j=1}^{N} w_{ij} \boldsymbol{y}_{jt}$. The w_{ij} denote pre-specified weights (we let $w_{ii} = 0$, $w_{ij} \geq 0$ and $\sum_{j=1}^{N} w_{ij} = 1$ for $i, j = 1, \dots, N$) that capture the strength of the linkages. The process in Eq. (4) is augmented by Q lags of these non-domestic cross-sectional averages $\boldsymbol{y}^*_{it}$, with associated $k \times k$ coefficient matrices $\boldsymbol{B}_{iq,t}$ $(q = 1, \dots, Q)$. The vector $\boldsymbol{\beta}_{it}$ associated with the log of the factor volatility h_t is of dimension $k \times 1$.

Our setup allows for interpreting $\boldsymbol{\beta}_{it}$ as the impact of uncertainty h_t on the endogenous variables of country i. We exploit this notion for calculating impulse response functions. Considering h_t to be orthogonal to the VAR errors implies that we do not impose restrictions on the contemporaneous effects, which relates to recursive identification schemes that order uncertainty indices first (see, e.g. Bloom, 2009). Empirical evidence for the credibility of this exogeneity assumption is provided by Carriero *et al.*

[4] In the empirical application, the likelihood turns out to be quite flat for σ_h, and we therefore impose the restriction $\sigma_h = 0.2$. Evaluating various values for this parameter over a grid suggests this choice to be only of minor importance for the results.

(2019), who find little evidence for endogenous responses of macroeconomic uncertainty to movements in key macroeconomic variables (see also Ludvigson *et al.*, 2015).

Before proceeding, we recast the model in standard regression form for notational simplicity,

$$\boldsymbol{y}_{it} = \boldsymbol{C}_{it}\boldsymbol{x}_{it} + \boldsymbol{\epsilon}_{it}, \tag{5}$$

with $\boldsymbol{x}_{it} = (1, \boldsymbol{y}'_{it-1}, \ldots, \boldsymbol{y}'_{it-P}, \boldsymbol{y}^{*\prime}{}_{it-1}, \ldots, \boldsymbol{y}^{*\prime}{}_{it-Q}, h_t)'$, and $\boldsymbol{C}_{it} = (\boldsymbol{\alpha}_{it}, \boldsymbol{A}_{i1,t}, \ldots, \boldsymbol{A}_{iP,t}, \boldsymbol{B}_{i1,t}, \ldots, \boldsymbol{B}_{iQ,t}, \boldsymbol{\beta}_{it})$. In what follows, it is convenient to consider the jth equation of country i in Eq. (5) which is given by

$$y_{ij,t} = \boldsymbol{C}'_{ij,t}\boldsymbol{x}_{it} + \epsilon_{ij,t}.$$

We refer to the jth row of the matrix $\boldsymbol{C}_{it}$ by $\boldsymbol{C}_{ij,t}$, which is a vector of dimension $\tilde{K}\times 1$ with $\tilde{K} = k(P+Q)+2$. The state vector is assumed to follow a random walk process

$$\boldsymbol{C}_{ij,t} = \boldsymbol{C}_{ij,t-1} + \boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Theta}_{ij}), \tag{6}$$

with diagonal $\tilde{K} \times \tilde{K}$ variance-covariance matrix $\boldsymbol{\Theta}_{ij} = \text{diag}(\theta_{ij,1}, \ldots, \theta_{ij,\tilde{K}})$.

As for the stochastic volatility specification, if $\theta_{ij,l}$ equals zero in Eq. (6), the respective coefficient is constant over time. To test the restriction $\theta_{ij,l} = 0$, we introduce the non-centered parameterization set forth by Frühwirth-Schnatter and Wagner (2010), which allows to impose standard shrinkage priors on these innovation variances. In particular, this approach splits the model coefficients into a constant and a time-varying part, a feature we exploit for designing sensible priors for the high-dimensional multivariate system proposed in this paper.

We proceed with rewriting the model in its non-centered parameterization.[5] Using a $\tilde{K}\times 1$-vector containing the square root of the state innovation variances in Eq. (6) denoted $\sqrt{\boldsymbol{\Theta}_{ij}} = \text{diag}(\sqrt{\theta_{ij,1}}, \ldots, \sqrt{\theta_{ij,\tilde{K}}})$, the reparameterized measurement equation is

$$y_{ij,t} = \boldsymbol{C}'_{ij,0}\boldsymbol{x}_{it} + \tilde{\boldsymbol{C}}'_{ij,t}\sqrt{\boldsymbol{\Theta}_{ij}}\boldsymbol{x}_{it} + \epsilon_{ij,t}. \tag{7}$$

Let $\tilde{c}_{ijl,t}$ denote a typical element of $\tilde{\boldsymbol{C}}_{ij,t}$, then the transformation $c_{ijl,t} = c_{ijl,0} + \sqrt{\theta_{ij,l}}\tilde{c}_{ijl,t}$ yields the corresponding state equation

$$\tilde{\boldsymbol{C}}_{ij,t} = \tilde{\boldsymbol{C}}_{ij,t-1} + \boldsymbol{v}_t, \quad \boldsymbol{v}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I}_{\tilde{K}}),$$

with $\tilde{\boldsymbol{C}}_{ij,0} = \boldsymbol{0}_{\tilde{K}}$. This procedure moves the square root of the innovation variances to the states into Eq. (7), implying that the measurement equation features all unknown parameters. The resulting state space representation has the convenient property that the $\sqrt{\theta_{ij,l}}$ can be treated as standard regression coefficients, and flexible shrinkage priors can be applied.

Stochastically selecting whether series should feature time-variation in their respective volatilities can be carried out using a transformation in similar vein (see Frühwirth-Schnatter and Wagner, 2010; Kastner and Frühwirth-Schnatter, 2014). Conditional on $\boldsymbol{L}\boldsymbol{f}_t$ and the full history of the VAR coefficients $\boldsymbol{C}_{it}$, we obtain a set of unrelated heteroscedastic error terms $\boldsymbol{\eta}_t$ by the diagonal structure of $\boldsymbol{\Omega}_t$. Here, we

[5]For applications of this approach in a VAR context see Feldkircher *et al.* (2017), Bitto and Frühwirth-Schnatter (2019) and Huber *et al.* (2019).

use $\eta_{ij,t}$ to indicate the error term of the jth equation for country i. Squaring and taking logs of $\eta_{ij,t}$ and using $\omega_{ij,t} = \sqrt{\sigma_{\omega ij}}\tilde{\omega}_{ij,t}$ results in

$$\tilde{\eta}_{ij,t} = \sqrt{\sigma_{\omega ij}}\tilde{\omega}_{ij,t} + v_{ij,t}, \quad v_{ij,t} \sim \ln \chi(1),$$
$$\tilde{\omega}_{ij,t} = \tilde{\omega}_{ij,t-1} + w_{ij,t}, \quad w_{ij,t} \sim \mathcal{N}(0, 1),$$

again moving the square root of the innovation variances $\sqrt{\sigma_{\omega ij}}$ from the state to the measurement equation. The transformation again allows to impose shrinkage priors on these coefficients, potentially pushing the model towards a homoscedastic specification if suggested by likelihood information.

2.2. *Prior distributions*

Bayesian methods are employed for estimation and inference. The panel structure of the data allows for constructing flexible shrinkage priors that are equipped to extract both cross-sectional information and moreover shrink the model towards sparsity, resulting in more precise inference. Before proceeding with the prior setup, it is necessary to stack the coefficients for the sake of notational simplicity. In particular, we use $\boldsymbol{c}_i = \text{vec}(\boldsymbol{C}'_{i1,0}, \ldots, \boldsymbol{C}'_{ik,0})$ to refer to the vector of constant regression coefficients associated with country i. In similar fashion, we collect square roots of the innovation variances $\sqrt{\theta_{ij,l}}$ in a vector $\sqrt{\boldsymbol{\theta}_i} = (\sqrt{\theta_{i1,1}}, \ldots, \sqrt{\theta_{i1,\tilde{K}}}, \ldots, \sqrt{\theta_{ik,1}}, \ldots, \sqrt{\theta_{ik,\tilde{K}}})'$. We index the jth element in $\boldsymbol{c}_i$ and $\sqrt{\boldsymbol{\theta}_i}$ by c_{ij} and $\sqrt{\theta_{ij}}$ respectively, with $j = 1, \ldots, k\tilde{K}$.

This article draws from the literature on the Bayesian treatment of panel data and global-local shrinkage priors. In particular, we center the prior on a common mean that is estimated from the data, reflecting the notion that macroeconomic dynamics across economies are typically similar. The prior setup thus relates to the random coefficients and heterogeneity model (Verbeke and Lesaffre, 1996; Allenby *et al.*, 1998; Frühwirth-Schnatter *et al.*, 2004), and restrictions often imposed in the context of panel VARs (see, for instance, Jarociński, 2010; Canova and Ciccarelli, 2013; Koop and Korobilis, 2016).

In what follows, we propose hierarchical priors akin to the Normal-Gamma (NG) shrinkage prior of Griffin and Brown (2010) recently adopted in the VAR context by Bitto and Frühwirth-Schnatter (2019) and Huber and Feldkircher (2019). Since an analogous setup is applied for different parts of the parameter space, we rely on the generic indicator $\bullet$ to indicate various combinations of indexes. For the constant part of the VAR coefficients, we assume that c_{ij} arises from

$$c_{ij}|\mu_{cj}, \tau_{cj} \sim \mathcal{N}(\mu_{cj}, \tau_{cj}), \quad \tau_{cj}|\lambda_c \sim \mathcal{G}(a_\bullet, a_\bullet\lambda_c/2), \quad \lambda_c \sim \mathcal{G}(d_{\bullet 0}, d_{\bullet 1}). \tag{8}$$

Here, a key novelty is that we do not push all country-specific coefficients towards zero, but rather towards a common mean μ_{cj}. The overall degree of shrinkage is determined by a global shrinkage parameter λ_c, thus serving as a general indicator of cross-country homogeneity. To provide flexibility for country-specific macroeconomic dynamics and deviations from the common mean, we introduce local scaling parameters τ_{cj}. In the presence of heavy shrinkage governed by λ_c, the τ_{cj} allow for flexibly selecting idiosyncrasies in coefficients across economies. This is an innovation compared to similar approaches (see, for instance, Malsiner-Walli *et al.*, 2016; Fischer *et al.*, 2019) who solely rely on a set of Gamma priors on these variances, disregarding a common degree of overall shrinkage towards homogeneity.

Shrinkage on the innovation variances of the states in Eq. (6) is introduced in similar vein. We follow Bitto and Frühwirth-Schnatter (2019) and stipulate a Gamma prior on these variances, which combined with a hierarchical prior relying again on Gamma distributions yields the setup they term the double Gamma prior. This is advantageous to the often employed inverse Gamma prior, because it does not artificially pull mass away from zero, a crucial feature when interest centers on stochastically shrinking the time-varying coefficients towards constancy. Frühwirth-Schnatter and Wagner (2010) show that this is equivalent to imposing a Gaussian prior on the square root of the state innovation variances,

$$\sqrt{\theta_{ij}}|\mu_{\theta j}, \tau_{\theta j} \sim \mathcal{N}(\mu_{\theta j}, \tau_{\theta j}), \quad \tau_{\theta j}|\lambda_\theta \sim \mathcal{G}(a_\bullet, a_\bullet \lambda_\theta/2), \quad \lambda_\theta \sim \mathcal{G}(d_{\bullet 0}, d_{\bullet 1}).$$

As in the case of the constant coefficients of the model, we introduce a common mean $\mu_{\theta j}$ rather than pushing the variances towards zero. This feature captures the notion that not only the constant coefficients across countries may be similar, but also the degree of time variation of the model parameters. The global shrinkage parameter λ_θ exerts shrinkage towards cross-sectional homogeneity of the innovation variances, while the local scalings $\tau_{\theta j}$ allow for flexibility and heterogeneity across countries governed by data information.

The first hierarchy of priors captures the notion that the dynamic coefficients of the model might be similar over the cross-section. However, VARs with drifting coefficients are prone to overfitting issues. We deal with this problem and induce sparsity in the coefficient matrices by imposing another NG prior to achieve regularization at the second level of the hierarchy. On the common mean μ_{sj} (for $s \in \{c, \theta\}$) we specify

$$\mu_{sj}|\tau_{\mu_s j} \sim \mathcal{N}(0, \tau_{\mu_s j}), \quad \tau_{\mu_s j}|\lambda_{\mu_s} \sim \mathcal{G}(a_\bullet, a_\bullet \lambda_{\mu_s}/2), \quad \lambda_{\mu_s} \sim \mathcal{G}(d_{\bullet 0}, d_{\bullet 1}).$$

This setup pushes the elements in the common mean towards zero, where the overall level of shrinkage is again governed by the global parameter λ_{μ_s}. Similar to the first prior hierarchy, the prior allows for non-zero elements if suggested by the data via the local scalings $\tau_{\mu_s j}$. This completes the setup for the VAR coefficients and the state innovation variances.

For the stochastic volatility specification we rely on analogous priors. In particular, for the state innovation variances of the stochastic volatility processes for the jth variable of country i, we impose Gamma distributed priors, translating to Gaussian priors on the square root of these variances. The prior is given by

$$\sqrt{\sigma_{\omega ij}}|\tau_{\sigma ij} \sim \mathcal{N}(0, \tau_{\sigma ij}), \quad \tau_{\sigma ij}|\lambda_\sigma \sim \mathcal{G}(a_\bullet, a_\bullet \lambda_\sigma/2), \quad \lambda_\sigma \sim \mathcal{G}(d_{\bullet 0}, d_{\bullet 1}),$$

with the global shrinkage parameter λ_σ pushing the model towards a homoscedastic specification. The local scalings $\tau_{\sigma ij}$ allow for non-zero state innovation variances. Intuitively, if $\tau_{\sigma ij}$ is small, we introduce substantial prior information and the parameter is pushed towards zero, ruling out time-variation in the respective volatility. For larger values of $\tau_{\sigma ij}$, the prior is less informative and allows for movements in the corresponding error variances.

It remains to specify prior distributions on the factor loadings in $\boldsymbol{L}$. Here, we stack the free elements in a vector $\boldsymbol{l}$ with typical element l_j for $j = 1, \ldots, R$ ($R = Kd - d(d+1)/2$)) and again opt for an NG shrinkage prior,

$$l_j|\tau_{Lj} \sim \mathcal{N}(0, \tau_{Lj}), \quad \tau_{Lj}|\lambda_L \sim \mathcal{G}(a_\bullet, a_\bullet \lambda_L/2), \quad \lambda_L \sim \mathcal{G}(d_{\bullet 0}, d_{\bullet 1}). \tag{9}$$

This choice implies shrinkage towards sparsity governed by the global parameter λ_L, while the local scalings τ_{Lj} once more serve to pull prior mass away from zero if likelihood information suggests non-zero factor loadings.[6]

Until now we remained silent on the choices of hyperparameter values. In the empirical specification, and referring by $\bullet$ to the indexes $\{c, \theta, \mu_s, \sigma, L\}$, we follow the literature and set $d_{\bullet 0} = d_{\bullet 1} = 0.01$ which implies heavy shrinkage via the global shrinkage parameter. Note that the hyperparameter $a_\bullet$ for the local scalings plays a crucial role in the specific properties of the prior. In fact, setting $a_\bullet = 1$ would yield the Bayesian LASSO (Park and Casella, 2008) used in Belmonte *et al.* (2014). Given that the generic prior is applied to a range of different quantities of the model's parameter space, we integrate out this hyperparameter by imposing exponential priors $a_\bullet \sim \mathcal{E}(1)$.

This completes the prior setup for achieving regularization in the high-dimensional state space model. Full conditional posterior distributions obtained from combining the likelihood function with the priors and the corresponding estimation algorithm are discussed in Appendices A and B. Fortunately, most of the distributions are of well-known form, allowing for a simple Markov chain Monte Carlo (MCMC) algorithm to obtain draws from the joint posterior using Gibbs sampling.

3. DATA AND MODEL SPECIFICATION

In this section, we introduce the dataset and discuss several important aspects in terms of model specification. Our dataset consists of monthly data for the period ranging from 1991:04 to 2018:07 for six economies: France (FRA), Germany (DEU), the United Kingdom (GBR), Italy (ITA), Japan (JPN), and the United States (USA). Consequently, the information set covers the G7 countries, similar to Crespo Cuaresma *et al.* (2017), except Canada due to limitations of government bond yield data.

Macroeconomic and financial quantities across countries included in the system are obtained from various sources. In particular, the model features series on industrial production (IP, as a monthly indicator of economic activity), unemployment (UN), year-on-year consumer price inflation (PR), exports (EX) and equity prices (EQ), downloaded from the FRED database of the Federal Reserve Bank of St. Louis. Industrial production, exports and equity prices enter the model in natural logarithms. To construct the cross-sectional weights for establishing links between economies, we rely on bilateral annual trade flows averaged over the sample period. Moreover, data on government bond yields at different maturities are downloaded from Quandl.[7]

A crucial determinant of business cycle fluctuations and the transmission of uncertainty shocks to the real sector of the economy are financial markets, with changes in term spreads being of particular importance (Gilchrist *et al.*, 2009; Gilchrist and Zakrajšek, 2012; Gilchrist *et al.*, 2014; Alessandri and Mumtaz, 2019). For a parsimonious representation of the full term structure of interest rates across countries, we adopt a Nelson-Siegel type model (see Nelson and Siegel, 1987; Diebold and Li, 2006). Government bond yield curves are estimated employing a factor model denoting yields by $r_{it}(\tau)$ at maturity τ,

$$r_{it}(\tau) = \mathfrak{L}_{it} + \mathfrak{S}_{it}\left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau}\right) + \mathfrak{C}_{it}\left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)\right). \tag{10}$$

[6] For a recent contribution proposing a comparable prior setup, see Kastner (2019).

[7] All series are available for download at fred.stlouisfed.org and quandl.com.

This setup allows the factors $\mathfrak{L}_{it}$, $\mathfrak{S}_{it}$ and $\mathfrak{C}_{it}$ to be interpreted as the level, (negative) slope and curvature of the yield curve, and may be estimated using ordinary least squares.[8] Using an $m \times 1$-vector of macroeconomic indicators $\boldsymbol{m}_{it}$, we exploit the yield curve fundamentals extracted in Eq. (10) to construct the $k \times 1$ endogenous vector $\boldsymbol{y}_{it} = (\boldsymbol{m}'_{it}, \mathfrak{L}_{it}, \mathfrak{S}_{it}, \mathfrak{C}_{it})'$ for $t = 1, \dots, T$ specific to country $i = 1, \dots, N$. In the discussion of the empirical results, $\mathfrak{L}_{it}$, $\mathfrak{S}_{it}$ and $\mathfrak{C}_{it}$ are labeled NSL, NSS and NSC, respectively.

All dimensions of the involved vectors can be derived based on $k = 8$ and $N = 6$. To select the lag order of the model and the number of latent factors that drive the full system variance covariance matrix, we rely on the deviance information criterion (DIC, Spiegelhalter *et al.*, 2002). This measure allows for model comparison and establishes a trade-off between model fit and complexity. We estimate the model over a grid of lag and latent factor combinations, and choose the specification minimizing the DIC. To add to the robustness of our findings, we iterate this procedure a number of times for all specifications and calculate the empirical standard deviation of the DIC. This procedure selects a model with $P = Q = 2$ lags and $d = 4$ factors.

For the empirical application, we slightly adopt the general prior setup put forward in Section 2. In particular, to reduce influence of the prior setup on the estimated impact of uncertainty, we use a rather diffuse prior on the constant part of these coefficients with prior variance equal to ten. The square roots of the state innovation variances of the impact vector are tightly centered on zero a priori. The latter choice mutes differences in impact reactions over time, but improves the stability of the model.

4. EMPIRICAL RESULTS

In the following results, we examine the consequences of international uncertainty shocks for the set of six economies. First, we identify similarities and idiosyncrasies across countries. Second, we discuss our measure of international uncertainty and link it to established proxies. Finally, we provide a thorough discussion of the dynamic responses for the macroeconomic and financial variables to uncertainty shocks.

4.1. *Homogeneity and heterogeneity across countries and over time*

In this section, we illustrate the key features of the proposed prior setup in terms of homogeneities and heterogeneities across countries and over time. In a first step, we assess the degree of sparsity imposed on the common mean that is inferred from the country-specific models. As a second step, we assess differences in coefficients across countries by analyzing the amount of shrinkage of country-specific coefficients towards the common mean.

Shrinkage towards sparsity

The non-centered parameterization of the state space model allows to investigate both shrinkage on the common mean of the time-invariant part of the VAR coefficients μ_{cj}, and the corresponding state innovation variances $\mu_{\theta j}$. Here, shrinkage is governed by the scaling parameters $\tau_{\mu_c j}$ and $\tau_{\mu_\theta j}$. Figure 1 shows the respective posterior mean of this variable on the logarithmic scale. Panel (a) indicates $\log(\tau_{\mu_c j})$, scalings associated with the constant part of the VAR coefficients, while (b) depicts $\log(\tau_{\mu_\theta j})$ associated

[8]We adopt a two-stage procedure to reduce the computational burden in the empirical application. The factor loadings are determined by the parameter $\lambda = 0.0609$ (see Diebold and Li, 2006, for details on this choice). For a more detailed discussion of how the three factors relate to level, slope and curvature of the yield curve, see also Diebold *et al.* (2006).

(a) Time-invariant VAR coefficients **(b)** Square root state innovation variances

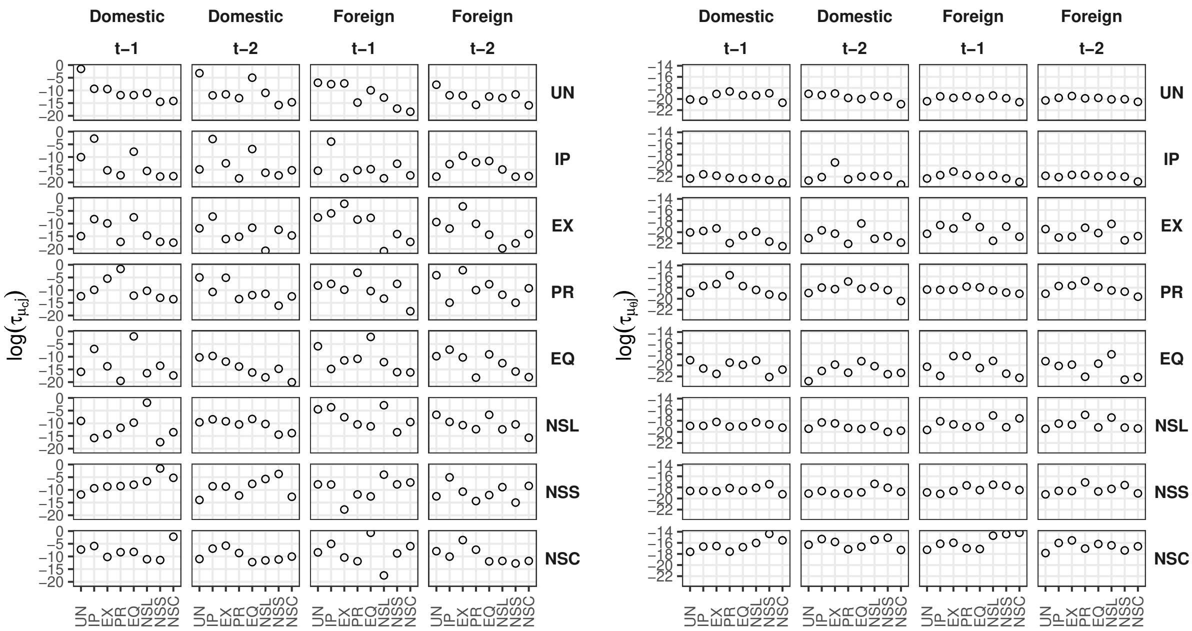

Fig. 1: Log posterior mean of prior variances $\tau_{\mu_s j}$ shrinking the common mean to zero.

Note: Panel (a) shows the prior variances associated with the common mean of the constant part of the VAR coefficients μ_{cj}, while panel (b) depicts the prior variances associated with the common mean of the state innovation variances in $\mu_{\theta j}$. The columns refer to the coefficients associated with a countries' own lagged variables in $\boldsymbol{y}_{it-p}$ (labeled "Domestic") of lag $t-p$, while "Foreign" indicates the coefficients associated with $\boldsymbol{y}^*_{it-q}$ at $t-q$. Variables (rows): Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve.

with the state innovation variances. Smaller values indicate heavier shrinkage towards zero. Note that due to visualization purposes and the imposed prior restrictions, we do not present the corresponding prior variances for the intercept term and the impact vector $\boldsymbol{\beta}_{it}$.

The first column of Fig. 1(a) highlights the first own lag of each equation in μ_{cj} to feature mainly non-zero coefficients, reflected in values of $\log(\tau_{\mu_c j})$ close to zero. This implies that only little shrinkage towards zero is imposed on these coefficients by the resulting loose prior variance $\tau_{\mu_c j}$. Such patterns, albeit less distinctive, are also observable for the second lag of the domestic coefficients in the second column. However, we generally detect tighter prior variances for the second lags, with differences depending on the respective equation. The equity price equation, for instance, and to a slightly lesser degree the equations associated with the Nelson-Siegel factors, exhibit tighter shrinkage governed by $\tau_{\mu_c j}$. Two equation specific idiosyncrasies are worth mentioning. First, both the first and second lag of equity prices feature less shrinkage for the unemployment, export, and especially the industrial production equations. Second, the second lag of unemployment appears to be crucial in the inflation equation, pointing towards a Phillips curve type relationship.

Turning to the third and fourth columns that indicate shrinkage on the foreign lags per equation, we find similar shrinkage patterns when comparing to the first domestic lag. Interestingly, non-domestic movements appear to play a role in the dynamic evolution of the Nelson-Siegel factors. In general, the results point towards the necessity of considering international dynamics, a feature explicitly addressed by the proposed multi-country approach.

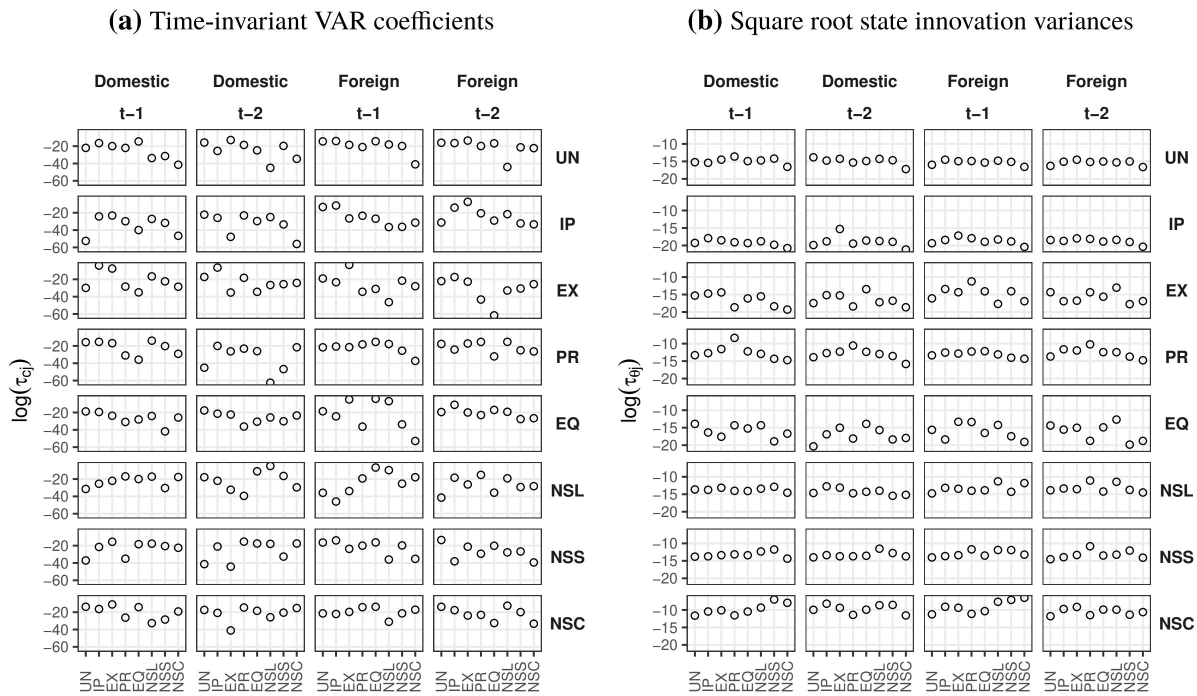

Fig. 2: Log posterior mean of prior variances τ_{sj} shrinking country-specific coefficients towards μ_{sj}.

Note: Panel (a) shows the prior variances associated with the constant part of the VAR coefficients c_{ij}, while panel (b) depicts the prior variances associated with the square root of state innovation variances $\sqrt{\theta_{ij}}$. The columns refer to the coefficients associated with a countries' own lagged variables in $\boldsymbol{y}_{it-p}$ (labeled "Domestic") of lag $t-p$, while "Foreign" indicates the coefficients associated with $\boldsymbol{y}^*_{it-q}$ at $t-q$. Variables (rows): Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve.

Figure 1(b) provides evidence of shrinkage towards zero of the state innovation variances that drive time-variation in the model coefficients. Note, however, that shrinkage of the common mean towards zero does not necessarily imply constant model coefficients, due to additional flexibility on the second prior hierarchy. A key finding is that the unemployment and industrial production equations are pushed strongly towards a constant parameter specification both for domestic and foreign lags. A simliar picture is present in the inflation equation, albeit at a slightly lower overall degree of shrinkage induced by the respective $\tau_{\mu_\theta j}$, and for the Nelson-Siegel level and slope factors. The higher value of $\log(\tau_{\mu_\theta j})$ on the first own domestic lag of inflation in the inflation equation suggests changes in the persistence of prices over time. Even more variation across the lags of variables is present for exports, equity prices and the Nelson-Siegel curvature factor equations. The overall least degree of shrinkage is imposed for the Nelson-Siegel curvature factor, implying a substantial degree of time-variation in the respective equation.

Shrinkage towards cross-sectional homogeneity

Next, we analyze the estimated prior variances τ_{cj} and $\tau_{\theta j}$ that shrink country-specific coefficients towards μ_{cj} and $\mu_{\theta j}$, respectively. Again, we consider the posterior mean of $\log(\tau_{cj})$ and $\log(\tau_{\theta j})$ in Fig. 2. The scalings provide a natural measure of similarity across countries. Values close to zero (or large negative numbers on the log-scale) yield a situation referred to as cross-sectional homogeneity in the panel literature (see Canova and Ciccarelli, 2013). Here, coefficients in the country-specific equations are strongly pushed towards the common mean. In the adverse case of looser priors, we observe a situation where macroeconomic dynamics potentially differ across countries.

One notable result in Fig. 2(a) is that all coefficients are strongly pushed towards homogeneity, suggested by predominantly large negative values for $\log(\tau_{cj})$. No clear patterns of similarities are visible across equations or both the domestic and foreign lag structure, and we thus proceed with results in the context of equation-specific shrinkage. Note that the first own domestic lags per equation usually feature less heavy shrinkage towards the common mean (except for inflation and equity prices), implying subtle differences in the persistence of the considered series across countries. Particularly strong evidence of homogeneity is present for subsets of domestic and foreign lags in all equations.

Figure 2(b) displays that heavy shrinkage on the state innovation variances is applied to all domestic and foreign lags in the unemployment and industrial production equation. A similar picture emerges for the inflation equation, and the dynamics captured in the context of the Nelson-Siegel level and slope factors. However, some variables appear to require flexibility in terms of country-specific breaks in the coefficients. Substantial differences in the amount of shrinkage towards homogeneity across domestic and foreign lags are featured in the export and equity price equations. The least degree of homogeneity is apparent in the context of the dynamic relationships between the Nelson-Siegel curvature factor and the remaining variables in the model.

Combining the discussions in the context of Figs. 1 and 2 allows for different scenarios in terms of homogeneity across countries and the degree of induced sparsity: First, there is the possibility of heterogeneous non-zero coefficients and state innovation variances, in cases where both $\tau_{\mu_s j}$ and τ_{sj} are comparatively large. Here, prominent examples are provided by most first own lags of the domestic coefficients in their respective equation. Second, if both $\tau_{\mu_s j}$ and τ_{sj} are small, the prior setup implies heavy shrinkage of the country-specific parameters towards zero, for example regarding most state innovation variances in the equations for unemployment and industrial production. Third, for large $\tau_{\mu_s j}$ and small τ_{sj}, the prior implies homogeneous non-zero parameters featured mainly in the context of the first autoregressive foreign lags.

Interestingly, while no clear relationship between Fig. 1 and Fig. 2 in terms of the constant part of the VAR coefficients can be identified, the adverse is true for the state innovation variances. This implies that if the common mean of the latter is non-zero on the first hierarchy of the prior, this is typically associated with less heavy shrinkage towards the common mean on the second hierarchy of the prior. Supplementary results for the country-specific square roots of the state innovation variances and the unconditional mean per equation are reported in Appendix C.

Summarizing this section, three points are worth noting. First, shrinkage of the common mean towards sparsity differs substantially depending on the respective equation, the lag order and whether domestic or foreign parameters are considered. Second, a substantial part of the parameter space of the model is shrunk heavily towards cross-sectional homogeneity, indicating similarity of macroeconomic dynamics for the economies considered. Third, in light of the discussion relating to shrinkage on the state innovation variances, a key finding of this article is that evidence for time-variation in the VAR coefficients is limited.

Our results corroborate previous studies indicating that considering stochastic volatility usually suffices for adequately capturing nonlinear dynamics in macroeconomic datasets (see Sims and Zha, 2006; Aastveit *et al.*, 2017; Chan and Eisenstat, 2018). The proposed model detects this data-feature and stochastically shrinks the parameter space towards the more parsimonious specification. However, breaks in macroeconomic dynamics are not ruled out by the prior setup. Subtle nonlinearities in model parameters may be crucial in forecast exercises, and potentially yield illuminating patters in structural inference.

4.2. *The measure of uncertainty*

We proceed with a discussion of the obtained measure of uncertainty, depicted in Fig. 3. This figure shows the log-volatility h_t of the factors that enters the mean of the VAR process. The most striking episode of high international uncertainty occurs during the global financial crisis and subsequent economic downturn – the Great Recession. During this period, volatilities of the common factors are more than twice as high than at the second highest peak.

Several less pronounced episodes of similarly elevated levels of international uncertainty are worth noting. Chronologically, uncertainty rises in the first half of 1997, related to the Asian financial crisis. A spike in late 1998 reflects the Russian financial crisis and the subsequent collapse of the U.S. hedgefund Long-term Capital Management. Afterwards, a brief period of lower uncertainty is observable, coming to an end with the burst of the Dot-com bubble and the 9/11 terror attacks in late 2001. Sustained elevated levels, albeit declining, are observable until the end of 2003, a period encompassing the outbreak of the second Gulf War. The period between 2004 and the bancruptcy of the U.S. investment bank Lehman Brothers features relatively low levels of international uncertainty.

Surging international volatilities are detected by the model starting in late 2007, capturing the onset of the crisis in the U.S. subprime mortgage market and first signs of disturbances on credit markets. After a decline of common volatilities to pre-crisis levels around 2010, the second highest peak of h_t occurs in 2011, related to events during the European sovereign debt crisis. This period of elevated uncertainty sustains until late 2013. The most recent episode of high uncertainty emerges in early 2016, indicating peaks related to the Brexit referendum and the election of Donald Trump as President of the United States in late 2016.

Following this brief discussion of the measure in light of uncertainty-related events, we compare our findings to commonly adopted proxies for uncertainty. The set of measurements is obtained from various sources. We consider the geopolitical risk (GPR) index described in Caldara and Iacoviello (2018), the global policy uncertainty (GEPU) index and the world uncertainty index (WUI) constructed as described in Baker *et al.* (2016), and complement these international measures of uncertainty with the proxy employed in many empirical studies of uncertainty, the Chicago Board Options Exchange volatility index (VIX).[9] Moreover, we take the arithmetic average for all benchmark indices and label the resulting series “Mean” in corresponding visualizations. To make the scales of the uncertainty measurements comparable, we standardize all measures to lie in the unit interval.

The resulting series are depicted in Fig. 4. A few points are worth noting. First, h_t provides a smoother estimate of uncertainty. However, most peaks apparent in the benchmark uncertainty measures are traced accurately. Differences occur mainly in the magnitude of the implied level of uncertainty. For instance, “Mean” peaks in 2003, with most benchmark measures showing substantial uncertainty around the outbreak of the second Gulf War. The endogenous measure of uncertainty traces this peak, but at a comparatively lower level. The Great Recession peak in late 2008 on the other hand, exhibiting a spike

[9]The indices are available for download at www2.bc.edu/matteo-iacoviello/gpr.htm (GPR), policyuncertainty.com (GEPU and WUI) and the FRED database of the Federal Reserve Bank of St. Louis (VIX).

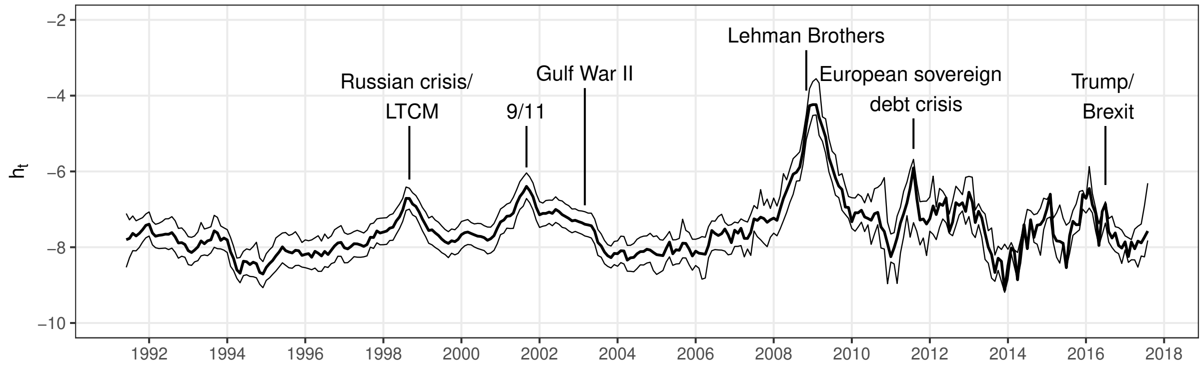

Fig. 3: Measurement of uncertainty depicting the log-volatility h_t of the common factors.

Note: The thick black line depicts the posterior median, alongside the 16th and 84th posterior percentiles (thin lines). *Russian crisis/LTCM* refers to the Russian crisis of 1998 and the resulting collapse of the U.S. hedge fund Long-term Capital Management, *9/11* indicates the terror attacks of September 11, 2001; *Lehman Brothers* refers to the bancruptcy filing of the investment bank Lehman Brothers in September 2008, and *Trump/Brexit* marks the election of Donald Trump as President of the United States and the Brexit referendum in the United Kingdom in mid/late 2016.

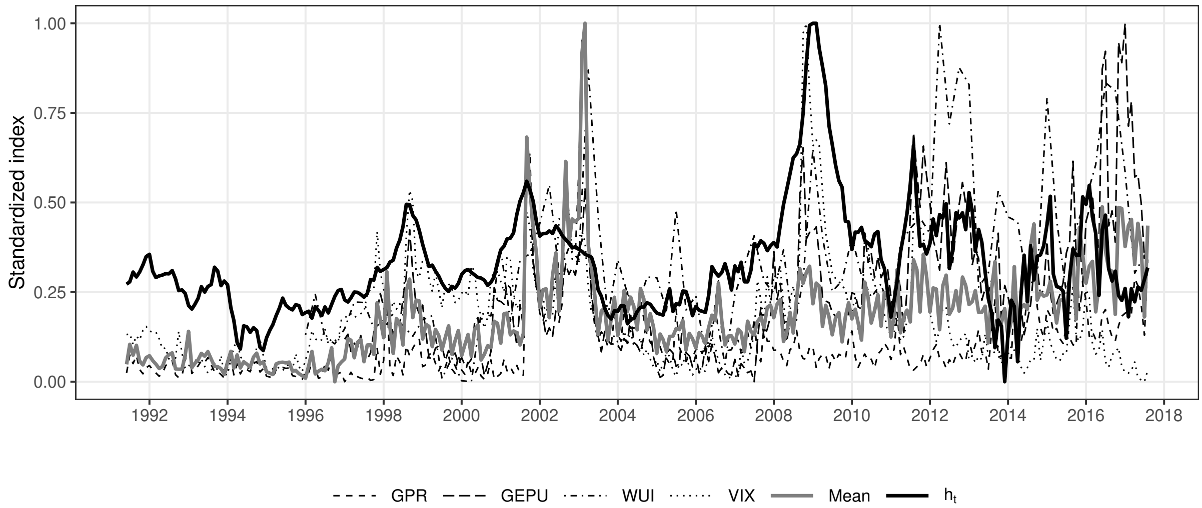

Fig. 4: Comparison of standardized uncertainty measures over time.

Note: Measures are standardized to lie in the unit interval. The thick black line depicts the posterior median of h_t. The remaining uncertainty measures are geopolitical risk (GPR), global policy uncertainty (GEPU), world uncertainty index (WUI), the Chicago Board Options Exchange volatility index (VIX), and the thick grey line refers to the arithmetic average of GPR, GEPU, WUI and VIX.

in the VIX and most other measures apart from GPR, is the highest level of uncertainty detected by h_t. Maximum values of WUI are associated with elevated levels in h_t, and also the peaks of GPR and GEPU coincide with upward movements in h_t. Besides the measures shown in Fig. 4, it is worth mentioning that our uncertainty measurement compares well to similar approaches dealing with the endogenous measurement of uncertainty (see Crespo Cuaresma *et al.*, 2017; Carriero *et al.*, 2018a).

Discussions of the evolution of common international uncertainty are complemented by the findings for idiosyncratic volatility series. Recall that the prior setup imposes shrinkage on the idiosyncratic residual variances towards constancy. As evidenced by the figure, the likelihood strongly suggests the necessity of a stochastic volatility specification. Hence, we refrain from a detailed discussion of the associated shrinkage parameters $\tau_{\sigma ij}$. It is worth mentioning that heteroscedasticity plays only a minor

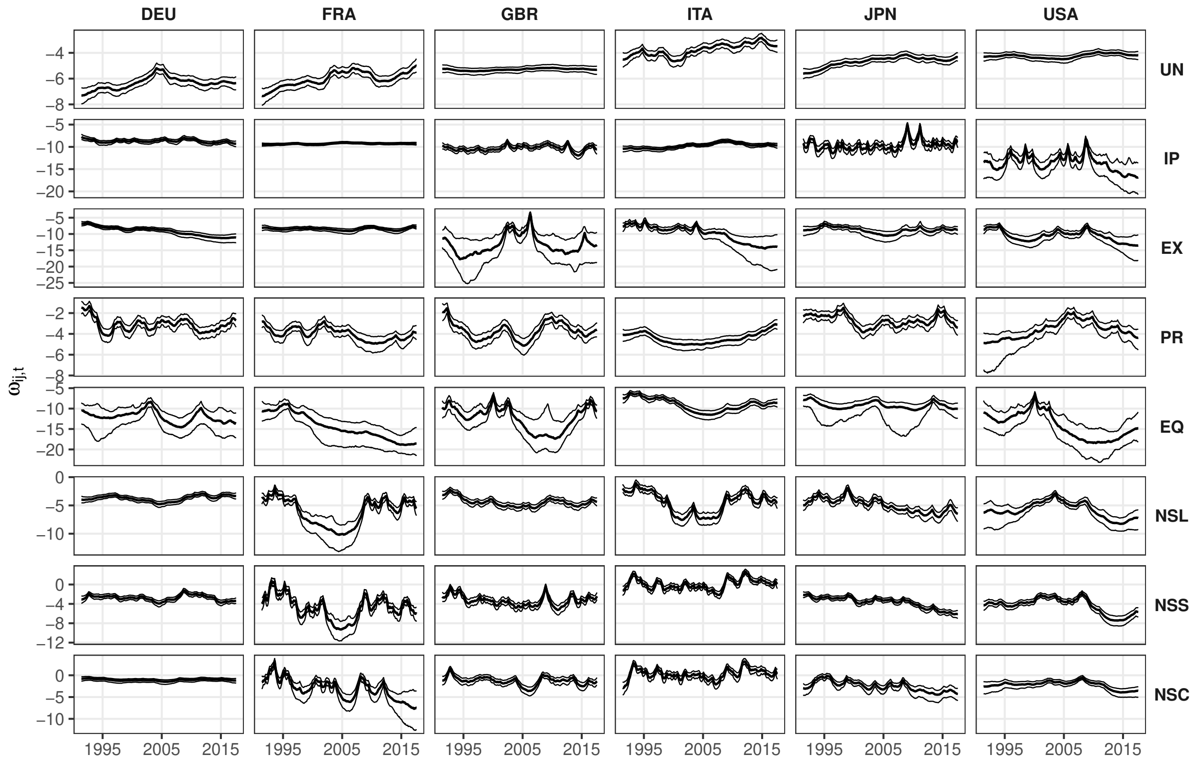

Fig. 5: Series-specific log-volatilities $\omega_{ij,t}$ for all variables across countries.

Note: The thick black line depicts the posterior median, alongside the 16th and 84th posterior percentiles (thin lines). Countries (columns): Germany (DEU), France (FRA), United Kingdom (GBR), Italy (ITA), Japan (JPN), United States (USA). Variables (rows): Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson and Siegel (1987) factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve.

role for a subset of the considered series, most prominently in the context of industrial production and unemployment for selected economies. The resulting log volatilities are shown in Fig. 5. Note that neither of these series enters the mean of the VAR process as in Mumtaz and Surico (2018) due to our focus on the effects of international uncertainty, however, they may be considered as a measurement of specific types of uncertainty. For instance, log volatilities associated with equity prices may be interpreted as country-specific financial market uncertainty. Individual series feature pronounced heterogeneities both in terms of the magnitude and the timing of peaks. This provides evidence that the approach employed for measuring common uncertainty in this paper discriminates well between country-specific events and international uncertainty-related events of significance.

Largest differences in the magnitude of the volatilities are visible for unemployment, with Germany and France exhibiting lower residual variances, when compared for instance to Italy or the United States. However, both feature substantial higher-volatility periods in the years surrounding 2005. While $\omega_{ij,t}$ for industrial production is rather homogenous for the continental European countries, the series of the remaining economies exhibit heterogeneities both in terms of magnitude and time-variation. The same is true, even though to a slightly lesser degree, in the case of export volatilities. Moreover, pronounced time-variation is clearly featured in the respective series relating to country-specific inflation dynamics, and equity prices. Volatilities associated with the factors capturing yield curve dynamics show marked similarities across countries, reflecting international commonalities in equity markets. This concludes the section on the measurement of uncertainty.

4.3. *Dynamic responses to uncertainty shocks*

In this section, we assess the dynamic responses across countries to an international uncertainty shock. With h_t entering the mean of the process, impulse response functions are computed based on the contemporaneous impact vector $\boldsymbol{\beta}_{it}$. This identification corresponds to ordering the uncertainty variable first in VARs achieving identification via zero-impact restrictions.

Figure 6 displays an overall summary of the dynamic responses for the periods between January 1992 and July 2017 on a biannual frequency, and reports the posterior median of the impulse response functions to the uncertainty shock. Colors refer to the respective period (red indicates early parts of the sample, blue marks later periods). Figure 7 depicts cumulative responses at the five year horizon. To save space, numerical values for peak and cumulative effects are provided for three selected periods in Tables 1 and 2: The first in the beginning of the sample (January 1993); the second in the middle period just before the Great Recession in a period of comparative stability (July 2004); and the third after the Financial Crisis of 2008/09 and the Great Recession (July 2017) at the end of the sample. Numbers in parentheses indicate 16th and 84th credible intervals alongside the posterior median.[10] Units are scaled as percentages for industrial production, exports and equity prices, while consumer price inflation, unemployment and the Nelson and Siegel (1987) factors for level, slope and curvature are in basis points (BPs).

Shrinkage is imposed via the prior setup on time-variation of the impact vector $\boldsymbol{\beta}_{it}$. This is reflected in time-invariant impact responses for all periods considered. In general, our results corroborate empirical findings from previous contributions, and both directions and magnitudes of the responses are similar. One notable result concerning the timing of the responses is that most react strongly on impact of the shock. We find significant increases of unemployment in all countries, while industrial production, exports, inflation and equity prices decrease. Timing and shape of the impulse responses for Nelson-Siegel level, slope and curvature factors indicate a flattening of the yield curve associated with overall decreases in interest rates at most maturities. In what follows we discuss our findings in detail, paying particular attention to country-specific dynamics and differences in transmission channels over time.

Unemployment. For unemployment reactions to international macroeconomic uncertainty shocks, we detect significant peaks on impact, ranging from two BPs in the case of Germany, France, the United Kingdom and Italy, while Japan exhibits larger magnitudes up to roughly four BPs. The largest unemployment responses result in the United States, with increases up to eight BPs roughly in line with Carriero *et al.* (2018a). The estimated effects are rather persistent, with significant positive reactions in terms of the the posterior median over the impulse response horizon of five years. Figure 6 suggests only a minor degree of time-variation, with the impacts leveling out slightly quicker in later parts of the sample. A key difference to previous findings in the literature is that unemployment effects peak on impact, and peter out slowly over the considered horizon, opposed to the often observed hump shaped impulse response functions (see, for instance, Carriero *et al.*, 2018b).

Closer inspection of time-variation of the cumulative effects over five years in the first row of Fig. 7 yields some interesting insights. Slight systematic decreases in the overall consequences of international uncertainty shocks on unemployment are visible for France, Italy, the United Kingdom and Japan. This notion is most pronounced for the United Kingdom, where cumulative effects decline from a significant 70.9 BPs in January 1992 to insignificant estimates of 44.3 BPs in July 2017. Different behavior occurs in Germany and the United States, with substantially larger cumulative responses at 112.1 BPs for Germany

[10] Additional results are available from the author upon request.

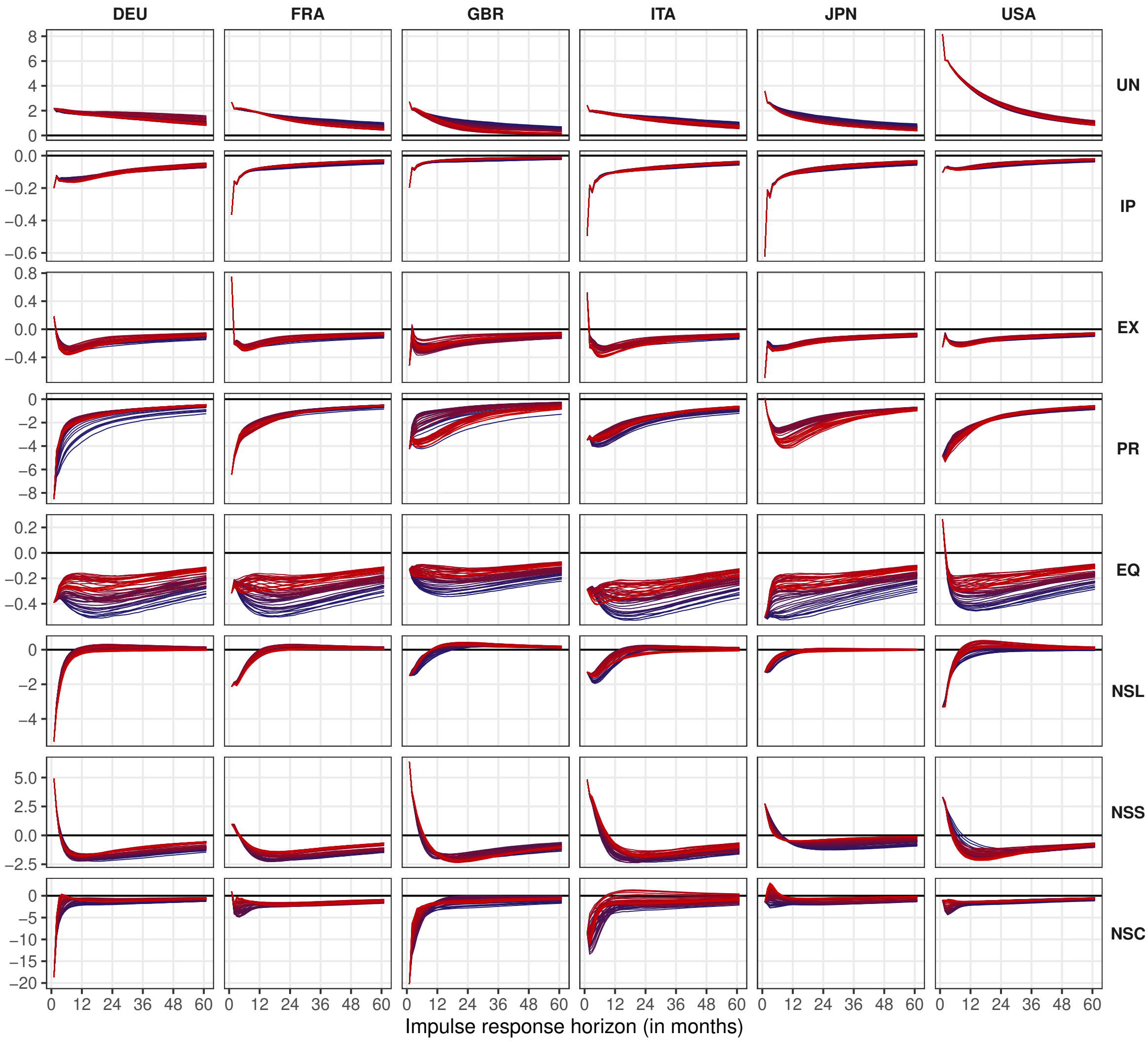

Fig. 6: Impulse response functions for selected periods to an international uncertainty shock.

Note: Posterior median of the impulse response functions over time, with the shading referring to the respective period: —— 1992:01 to —— 2017:07 on biannual frequency. The black line marks zero. Countries (columns): Germany (DEU), France (FRA), United Kingdom (GBR), Italy (ITA), Japan (JPN), United States (USA). Variables (rows): Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve.

in the years surrounding 2005, reflecting labor markets under severe stress during this period. For the U.S., estimates gradually amplify before the global financial crisis, with substantially larger effects close to 160 BPs during the Great Recession. This finding is mainly driven by higher persistence of the effects during this period.

Industrial production. Industrial production shows the largest declines in Italy and Japan, with significant negative peak responses on impact of 0.5 and 0.6 percent, respectively. The remaining countries exhibit rather homogeneous responses, with largest effects in France, followed by the United Kingdom and Germany of approximately 0.2 percent. The United States shows the smallest effects, with an approximate decline of 0.1 percent on impact. Note that for some countries posterior credible sets of the peak responses include zero. Time variation at a first glance again appears limited, however, as reported in the second row Fig. 7, subtle changes in the persistence of the estimated uncertainty shocks translates to time-varying patterns in terms of cumulative responses.

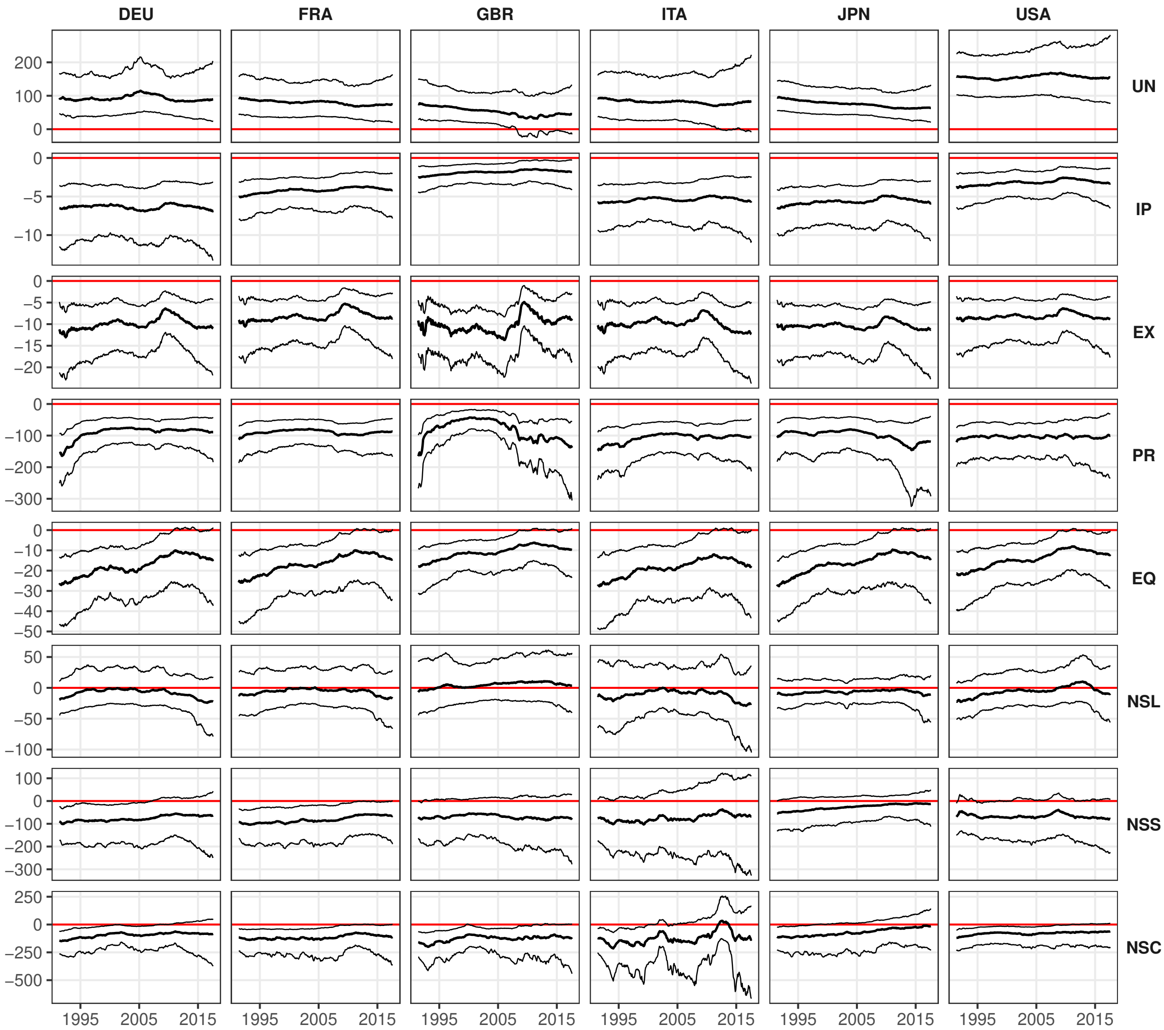

Fig. 7: Cumulative impulse response functions over time to an international uncertainty shock.

Note: The thick black line depicts the posterior median, alongside the 16th and 84th posterior percentiles (thin lines). The red line marks zero. Countries (columns): Germany (DEU), France (FRA), United Kingdom (GBR), Italy (ITA), Japan (JPN), United States (USA). Variables (rows): Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve.

Similar to our findings in the context of unemployment responses, cumulative effects for industrial production appear to gradually decrease in the first years of the sample period, in line with Mumtaz and Theodoridis (2018). However, except for the United Kingdom, and different to the homogeneous dynamics of cumulative responses for unemployment on the country-level, this trend disappears just prior to the outbreak of the global financial crisis, with larger resulting estimates. Interestingly, in a brief period after the Great Recession, uncertainty shocks appear to play a less important role for industrial production, a notion that reverts later in the sample. Considering the three selected periods, cumulative industrial production responses differ markedly over the cross-section, ranging from 6.3 percent (in Germany) to 2.4 percent (in the United Kingdom) in January 1992, decreasing slightly towards the end of the sample for most economies.

Exports. Exports indicate insignificant impacts close to zero for Germany and the United Kingdom, with a significant peak decline around two quarters after impact of roughly −0.3 percent. France and Italy exhibit positive impacts, but the responses quickly turn negative, with peak negative effects lying between −0.2 and −0.4 percent. Substantial decreases are indicated for the United States, and more strikingly, Japan, with decreases of about 0.5 percent in exports on impact which coincides with the peak response. Note that the magnitude of the estimates is approximately in line with findings by Crespo Cuaresma *et al.* (2017).

Regarding time-variation in the responses, no clear pattern emerges in terms of selected time points or the reported responses in Fig. 6, and no conclusions can yet be drawn whether the impact of uncertainty shocks changed systematically over time. Here, we again resort to Fig. 7, with the third row providing evidence of substantial differences over time in the cumulative responses of uncertainty shocks to exports. No clear pattern emerges previous to the global financial crisis, with estimates fluctuating approximately around −10 percent across countries. The largest fluctuations in the cumulative responses are observable for the United Kingdom. Analogous to the results for unemployment and industrial production, the consequences of uncertainty shocks on exports in the aftermath of the Great Recession are muted in comparison to previous periods. From 2015 onwards, the cumulative responses are again similar to earlier in the sample, with estimates of roughly −10 percent with minor differences over the cross-section.

Consumer price inflation. Our findings for inflation require a more thorough discussion. First, note that the employed index includes food and energy prices. Previous studies often use richer information sets comprised of multiple inflation indices. Second and relatedly, Fernández-Villaverde *et al.* (2015) identify two contradicting channels how uncertainty affects consumer prices: The so-called aggregate demand channel, characterized by reducing the consumption of households and thereby leading to an overall decrease in prices; and the upward-pricing bias channel, which yields increases in inflation based on profit-maximizing firms. In our case, the former appears to dominate the latter, with significant decreases of inflation on impact for most economies in row four of Fig. 7. The estimated peak effects for selected periods in Tables 1 and 2 indicate constancy in magnitudes ranging from −8.5 BPs in Germany on impact, to a mere −2.5 BPs for the case of Japan after two quarters (with insignificant impact responses).

The impulses for inflation in Germany, France, Italy and the United States exhibit only a small degree of persistence, with responses quickly leveling out. In terms of time-variation, the effects of uncertainty shocks on prices appear more persistent early in the sample, especially in Germany, and to a lesser degree in Italy. The shape of inflation responses in the United Kingdom is similar to the other countries before 2005. However, impulse response functions turn hump-shaped in later periods, comparable to those of Japan.

Further inspection of the estimates in light of Fig. 7 reveals substantial heterogeneities. First, we observe differences in posterior uncertainty over the sample period. Less precisely estimated cumulative effects mainly occur in the context of short-term interest rates hitting zero-lower bound for most economies, and we also detect differences in the posterior median for this period especially in the case of the United Kingdom. Second, inflated credible sets and differences in the posterior mean moreover occur early in the sample. Third, responses at the end of the sample period in July 2017 feature little cross-sectional heterogeneity, with cumulative estimates of approximately −100 BPs for most economies. Finally, idiosyncratic movements for the United Kingdom are worth mentioning. After large negative effects early in the sample, the consequences of uncertainty shocks on inflation declined substantially until late 2007. After the Great Recession, substantially larger effects are detected, as suggested by the evolution of the shape of the impulse responses for the United Kingdom in Fig. 6.

Table 1: Peak and cumulative impulse response functions for selected periods (DEU, FRA and GBR).

		Period *1992:01*			Period *2004:07*			Period *2017:07*		
Country	Variable	Peak	Hor	Cumulative	Peak	Hor	Cumulative	Peak	Hor	Cumulative
DEU	UN	2.2 (1.2, 3.2)	0	89.8 (40.0, 166.1)	2.2 (1.2, 3.2)	0	112.1 (50.9, 211.0)	2.2 (1.2, 3.2)	0	86.4 (25.9, 193.9)
	IP	−0.2 (−0.4, 0.0)	0	−6.3 (−11.6, −3.3)	−0.2 (−0.4, 0.0)	0	−6.8 (−11.3, −3.9)	−0.2 (−0.4, 0.0)	0	−6.7 (−12.6, −3.2)
	EX	−0.3 (−0.4, −0.1)	6	−11.6 (−21.1, −5.9)	−0.3 (−0.4, −0.2)	6	−10.7 (−17.6, −5.9)	−0.3 (−0.5, −0.2)	6	−10.7 (−20.8, −4.2)
	PR	−8.5 (−14.1, −2.9)	0	−138.6 (−225.1, −79.7)	−8.5 (−14.1, −2.9)	0	−78.3 (−128.3, −46.9)	−8.5 (−14.1, −2.9)	0	−89.7 (−171.5, −44.5)
	EQ	−0.5 (−0.8, −0.3)	15	−26.0 (−45.7, −12.8)	−0.4 (−0.5, −0.2)	0	−19.3 (−35.6, −8.3)	−0.4 (−0.5, −0.2)	0	−13.9 (−34.5, 0.1)
	NSL	−5.3 (−9.1, −1.5)	0	−14.6 (−39.3, 18.1)	−5.3 (−9.1, −1.5)	0	−5.7 (−28.5, 27.5)	−5.3 (−9.1, −1.5)	0	−22.3 (−76.4, 17.0)
	NSS	4.9 (0.5, 8.7)	0	−91.1 (−187.8, −23.2)	4.9 (0.5, 8.7)	0	−82.8 (−200.5, −6.4)	4.9 (0.5, 8.7)	0	−62.1 (−247.3, 33.0)
	NSC	−18.7 (−30.0, −6.2)	0	−139.2 (−290.8, −45.6)	−18.7 (−30.0, −6.2)	0	−95.8 (−230.6, −16.1)	−18.7 (−30.0, −6.2)	0	−85.9 (−346.0, 47.0)
FRA	UN	2.7 (1.4, 4.2)	0	89.2 (42.8, 160.9)	2.7 (1.4, 4.2)	0	83.2 (39.5, 145.8)	2.7 (1.4, 4.2)	0	73.6 (23.2, 156.6)
	IP	−0.4 (−0.5, −0.2)	0	−4.8 (−7.9, −2.9)	−0.4 (−0.5, −0.2)	0	−4.3 (−7.0, −2.6)	−0.4 (−0.5, −0.2)	0	−4.1 (−7.5, −2.0)
	EX	0.7 (0.3, 1.2)	0	−9.3 (−17.5, −4.1)	0.7 (0.3, 1.2)	0	−9.0 (−15.3, −4.7)	0.7 (0.3, 1.2)	0	−8.5 (−17.4, −2.9)
	PR	−6.4 (−10.9, −2.3)	0	−95.4 (−160.3, −57.3)	−6.4 (−10.9, −2.3)	0	−81.5 (−131.7, −48.2)	−6.4 (−10.9, −2.3)	0	−88.5 (−162.4, −47.3)
	EQ	−0.5 (−0.8, −0.2)	16	−25.0 (−44.4, −12.4)	−0.3 (−0.6, −0.1)	22	−16.9 (−31.5, −6.9)	−0.3 (−0.4, 0.2)	0	−13.9 (−33.2, −0.7)
	NSL	−2.1 (−3.8, −0.5)	0	−11.7 (−45.8, 23.3)	−2.1 (−3.8, −0.5)	0	−2.6 (−28.3, 30.0)	−2.1 (−3.8, −0.5)	0	−17.1 (−62.7, 25.5)
	NSS	−2.2 (−3.5, −0.8)	16	−103.0 (−195.7, −40.2)	−1.9 (−3.6, −0.5)	20	−86.6 (−192.5, −18.1)	−1.5 (−3.1, −0.3)	16	−63.2 (−172.4, −0.8)
	NSC	−3.5 (−6.3, −1.2)	3	−137.3 (−291.6, −44.3)	−3.1 (−9.1, 0.8)	1	−127.0 (−303.3, −33.0)	−2.2 (−5.4, −0.3)	21	−106.0 (−326.2, −1.4)
GBR	UN	2.7 (1.0, 4.6)	0	70.9 (27.1, 142.8)	2.7 (1.0, 4.6)	0	52.0 (14.4, 109.1)	2.7 (1.0, 4.6)	0	44.3 (−12.5, 123.7)
	IP	−0.2 (−0.3, −0.1)	0	−2.4 (−4.3, −1.1)	−0.2 (−0.3, −0.1)	0	−1.8 (−3.3, −0.8)	−0.2 (−0.3, −0.1)	0	−1.7 (−3.9, −0.3)
	EX	−0.5 (−0.6, −0.1)	0	−9.3 (−16.6, −4.4)	−0.5 (−0.6, −0.1)	0	−12.7 (−20.8, −7.6)	−0.5 (−0.6, −0.1)	0	−8.6 (−17.4, −2.9)
	PR	−4.3 (−8.7, 0.4)	0	−88.1 (−154.4, −45.7)	−4.3 (−8.7, 0.3)	0	−55.7 (−99.8, −22.6)	−4.3 (−8.7, 0.4)	0	−131.3 (−281.1, −57.1)
	EQ	−0.3 (−0.5, −0.2)	15	−16.7 (−29.0, −8.3)	−0.2 (−0.4, −0.1)	19	−11.1 (−20.8, −4.3)	−0.2 (−0.4, 0.0)	12	−9.4 (−22.3, 0.3)
	NSL	−1.5 (−4.6, 4.6)	0	−4.2 (−40.1, 47.4)	−1.5 (−4.6, 4.6)	0	6.7 (−19.8, 45.2)	−1.5 (−4.6, 4.6)	0	4.5 (−39.5, 54.2)
	NSS	6.4 (2.0, 10.7)	0	−75.8 (−197.2, 3.2)	6.4 (2.0, 10.7)	0	−60.0 (−171.9, 16.2)	6.4 (2.0, 10.7)	0	−73.8 (−249.4, 29.9)
	NSC	−20.2 (−34.3, −8.6)	0	−198.9 (−410.4, −72.4)	−20.2 (−34.3, −8.6)	0	−117.9 (−265.4, −35.2)	−20.2 (−34.3, −8.6)	0	−115.5 (−370.7, 5.2)

Notes: Countries shown are Germany (DEU), France (FRA), United Kingdom (GBR). Variables: Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve. *Peak* refers to the peak of the impulse response function in terms of the median, while *Hor* indicates the impulse response horizon when the peak occured in months (zero implying the impact response). *Cumulative* indicates cumulative impulse response functions over the full horizon. The numbers refer to the posterior median with the 16th and 84th percentile of the posterior distribution in parentheses.

Table 2: Peak and cumulative impulse response functions for selected periods (ITA, JPN and USA).

		Period *1992:01*			Period *2004:07*			Period *2017:07*		
Country	Variable	Peak	Hor	Cumulative	Peak	Hor	Cumulative	Peak	Hor	Cumulative
ITA	UN	2.4 (0.5, 4.4)	0	91.8 (33.5, 170.4)	2.4 (0.5, 4.4)	0	83.8 (27.4, 161.0)	2.4 (0.5, 4.4)	0	82.2 (−5.0, 212.6)
	IP	−0.5 (−0.7, −0.3)	0	−5.8 (−9.5, −3.5)	−0.5 (−0.7, −0.3)	0	−5.5 (−8.7, −3.3)	−0.5 (−0.7, −0.3)	0	−5.5 (−10.5, −2.4)
	EX	0.5 (0.3, 0.7)	0	−10.3 (−19.5, −4.8)	0.5 (0.3, 0.7)	0	−10.3 (−17.3, −5.4)	0.5 (0.3, 0.7)	0	−12.0 (−22.5, −4.8)
	PR	−3.8 (−6.4, −1.5)	2	−128.1 (−211.4, −72.6)	−3.5 (−7.3, −0.1)	0	−94.1 (−154.7, −56.1)	−3.5 (−7.3, −0.1)	0	−105.2 (−204.3, −51.3)
	EQ	−0.5 (−0.8, −0.3)	17	−26.3 (−47.4, −12.1)	−0.3 (−0.6, −0.1)	18	−18.4 (−34.4, −7.4)	−0.4 (−0.6, −0.2)	4	−17.2 (−41.1, −0.9)
	NSL	−1.8 (−3.6, 0.4)	3	−18.7 (−72.5, 38.9)	−1.4 (−3.2, 0.7)	1	−5.5 (−39.2, 31.9)	−1.6 (−3.3, 0.2)	3	−26.8 (−97.6, 28.3)
	NSS	4.8 (−3.9, 12.1)	0	−88.7 (−215.2, 12.0)	4.8 (−3.9, 12.1)	0	−79.9 (−240.5, 43.2)	4.8 (−3.9, 12.1)	0	−67.1 (−318.5, 115.5)
	NSC	−8.5 (−23.7, 6.4)	0	−170.0 (−380.7, −49.2)	−10.1 (−19.3, −2.0)	1	−145.9 (−416.3, 3.5)	−10.8 (−23.1, −1.4)	1	−121.7 (−585.5, 155.2)
JPN	UN	3.6 (2.2, 5.0)	0	90.9 (54.0, 142.8)	3.6 (2.2, 5.0)	0	74.6 (43.8, 122.3)	3.6 (2.2, 5.0)	0	64.4 (22.1, 126.3)
	IP	−0.6 (−0.9, −0.4)	0	−6.2 (−9.7, −3.8)	−0.6 (−0.9, −0.4)	0	−5.8 (−9.2, −3.7)	−0.6 (−0.9, −0.4)	0	−5.7 (−10.4, −3.0)
	EX	−0.7 (−1.2, −0.2)	0	−10.5 (−18.6, −5.4)	−0.7 (−1.2, −0.2)	0	−11.0 (−18.3, −6.5)	−0.7 (−1.2, −0.2)	0	−11.3 (−21.7, −5.0)
	PR	−2.5 (−3.9, −1.3)	6	−86.5 (−156.3, −45.6)	−2.5 (−4.1, −1.0)	4	−83.4 (−149.5, −42.3)	−3.5 (−5.9, −1.7)	8	−117.7 (−280.8, −43.9)
	EQ	−0.5 (−1.0, −0.1)	1	−25.5 (−42.4, −13.2)	−0.5 (−1.4, 0.3)	0	−16.3 (−30.2, −6.8)	−0.5 (−1.4, 0.3)	0	−13.2 (−34.2, 0.6)
	NSL	−1.3 (−2.5, 0.0)	1	−11.9 (−34.4, 11.2)	−1.3 (−3.0, 0.6)	0	−5.2 (−25.7, 14.4)	−1.3 (−3.0, 0.6)	0	−11.5 (−54.8, 17.7)
	NSS	2.7 (0.5, 4.9)	0	−49.0 (−125.6, 7.2)	2.7 (0.5, 4.9)	0	−23.7 (−91.5, 22.3)	2.7 (0.5, 4.9)	0	−12.0 (−100.6, 44.4)
	NSC	−2.1 (−5.1, 1.0)	2	−111.4 (−254.0, −17.6)	−1.6 (−4.4, 0.0)	24	−79.3 (−261.4, 11.8)	2.8 (−1.5, 6.9)	2	−11.9 (−210.3, 127.0)
USA	UN	8.1 (6.1, 10.2)	0	157.3 (100.8, 226.4)	8.1 (6.1, 10.2)	0	161.3 (102.1, 241.5)	8.1 (6.1, 10.2)	0	152.6 (80.5, 269.0)
	IP	−0.1 (−0.2, 0.0)	0	−3.7 (−6.3, −2.0)	−0.1 (−0.2, 0.0)	0	−3.3 (−5.3, −1.8)	−0.1 (−0.2, 0.0)	0	−3.2 (−6.2, −1.3)
	EX	−0.3 (−0.6, 0.1)	0	−8.7 (−16.5, −4.1)	−0.3 (−0.6, 0.1)	0	−8.7 (−14.6, −4.6)	−0.3 (−0.6, 0.1)	0	−8.6 (−16.9, −3.8)
	PR	−4.8 (−9.4, −0.4)	0	−109.3 (−190.1, −66.1)	−5.0 (−9.3, −0.9)	1	−104.7 (−173.4, −62.1)	−5.3 (−9.7, −0.9)	1	−102.0 (−223.7, −31.8)
	EQ	−0.5 (−0.7, −0.3)	13	−21.5 (−38.1, −10.9)	−0.3 (−0.5, −0.1)	10	−14.7 (−26.3, −6.2)	−0.3 (−0.5, −0.1)	7	−11.7 (−27.1, −1.3)
	NSL	−3.3 (−5.6, 0.6)	0	−20.0 (−48.6, 10.4)	−3.3 (−5.6, 0.6)	0	−5.9 (−29.0, 26.5)	−3.3 (−5.6, 0.6)	0	−9.6 (−54.1, 33.9)
	NSS	3.3 (−0.2, 6.4)	0	−54.7 (−140.3, 10.9)	3.3 (−0.2, 6.4)	0	−72.0 (−175.7, 4.7)	3.3 (−0.2, 6.4)	0	−78.1 (−222.5, 10.1)
	NSC	−3.4 (−6.2, −0.7)	2	−104.3 (−212.9, −37.0)	−3.4 (−6.2, −0.8)	2	−83.6 (−194.6, −19.5)	−1.3 (−3.3, 0.3)	6	−64.6 (−199.2, 6.7)

Notes: Countries shown are Italy (ITA), Japan (JPN), United States (USA). Variables: Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve. *Peak* refers to the peak of the impulse response function in terms of the median, while *Hor* indicates the impulse response horizon when the peak occured in months (zero implying the impact response). *Cumulative* indicates cumulative impulse response functions over the full horizon. The numbers refer to the posterior median with the 16th and 84th percentile of the posterior distribution in parentheses.

Equity prices. Equity prices, displayed in the fifth row of Fig. 7, prominently feature time-variation in the dynamic responses for all countries, with rather homogeneous patterns over the cross-section (except for the United Kingdom and the United States). The responses in the United Kingdom are less pronounced than in the other countries, with an impact of approximately −0.1 percent, peaking after roughly one to one and a half years at about −0.3 percent. For the United States, we observe an insignificant impact response quickly turning negative, with peak effects after one year between −0.25 and −0.5 percent, depending on the respective period. The remaining economies exhibit rather similar responses, with Japanese equity prices indicating the largest impact responses of around −0.5 percent. Germany, France and Italy show declines between −0.25 and −0.3 percent.

The cumulative responses for equity prices in the beginning of the sample period show large and significant homogeneous declines over the cross-section of economies between −26.3 and −21.5 percent, except for the United Kingdom where the posterior median is substantially smaller at −16.7 percent. The cumulative responses for equity prices move towards zero across all economies rather homogeneously for the period in the middle of the sample in July 2004, declining in absolute value by approximately 5 percentage points. In July 2017, the cumulative responses decline even further, with the 68 percent posterior credible set covering zero in some economies. However, note that substantial posterior mass is centered away from zero. A clear empirical regularity is that the impact of uncertainty shocks on equity prices declines over time, in line with findings provided in Mumtaz and Theodoridis (2018). However, note that this downward trend is not linear for the whole sample period, with subtle changes especially in periods associated with economic crisis and higher international uncertainty (see also Alessandri and Mumtaz, 2019; Bertolotti and Marcellino, 2019).

Nelson-Siegel factors. For interpretational clarity, recapture Eq. (10), where the loading on $\mathfrak{L}_{it}$ is a constant for all τ; hence it affects all maturities equally, and is interpreted as the long-term level of the yield curve. The loading associated with $\mathfrak{S}_{it}$ decreases rapidly in τ, and is thus closely related to the negative slope of the yield curve and term spreads (for details, see Diebold and Li, 2006). Consequently, an *increase* in $\mathfrak{S}_{it}$ implies a *decrease* in term spreads, and thus a flattening of the yield curve. Diebold *et al.* (2006) provide empirical evidence for the close relationship between this factor and central bank policy rates. The loading of $\mathfrak{C}_{it}$ is hump-shaped, and loads most strongly on the middle segment of the yield curve that affects its curvature.

Impulse responses for the Nelson-Siegel factors are displayed in the last three rows of Fig. 6. The dynamic evolution of the level factor exhibits substantial heterogeneity across countries, but appears comparatively constant over time with slight differences in the curvature of the responses. In particular, we find the largest and significant decreases on impact, coinciding with the peak response, for Germany of around −5.3 BPs. In general, the credible sets associated with the impulse responses of the level factor are rather large, and cover zero in most economies. The effects peter out quickly, with impulse responses returning to zero after about two quarters. Observed heterogeneity over the cross-section may originate from international capital flows toward safer assets in uncertain times (see, for instance, Caballero *et al.*, 2017). Figure 7 indicates that the posterior distribution of the cumulative effects for the level factor cover zero for all economies over the sample period considered, featuring detectable yet insignificant time-varying dynamics in the responses.

The slope factor detects significant positive reactions peaking instantaneously in Germany, the United Kingdom, and Japan. The effects for the remaining countries on impact are estimated less precisely, however, the posterior centers on positive values for all countries ranging from one to five BPs. An increase in the slope factors translates to a decrease in term spreads and a flattening of the yield curve, a phenomenon that has been linked to the emergence of recessions in the literature (Estrella and Mishkin, 1998). This effect reverses in subsequent months, turning significantly negative between one and one and a half years after impact across countries. Given the close empirical relationship between the slope factor and central bank policy, we conjecture that this pattern captures a delayed response of central banks, lowering policy rates to counteract detrimental economic effects of uncertainty shocks. Considering previous contributions, we hypothesize that the overall decrease in interest rates is thus related to expansionary monetary policy measures (both conventional and unconventional) enacted by central banks, and international capital flows towards safety.

Assessing cumulative effects, we find that estimates are statistically significant early in the sample for Germany and France, with decreases of approximately 90 to 100 BPs. The model captures large but insignificant effects for the remaining economies except Japan, which is unsurprising considering the country's recent monetary history. In general, the impact of uncertainty shocks on the slope factor appears to decrease over time, evidenced by subtle trends visible for most countries except the United Kingdom and the United States. At the end of the sample period, we do not observe significant cumulative effects for the countries considered.

Findings associated with the curvature factor signal decreases for most countries. Again, we observe pronounced heterogeneity over the cross-section, but also over time. The responses peak on impact for Germany and the United Kingdom at about 20 BPs, and approximately ten BPs in Italy. France, the United States and Japan show only small consequences of uncertainty shocks for middle-term maturities. Overall, this implies dynamics typically associated with a flattening of the yield curve. In terms of cumulative responses, we find systematic declines in the magnitude of the effects associated with inflated posterior uncertainty for Japan, dynamics that are also visible in the case of Germany, France, the United Kingdom and the United States. Minor differences occur for selected periods after the Great Recession. Italy presents a special case, with distinct periods featuring substantial differences in the cumulative responses. In particular, the estimated effects are much smaller during the early 2000s and the European sovereign debt crisis.

5. CONCLUDING REMARKS

This paper investigates the time-varying effects of international uncertainty shocks on macroeconomic and financial variables for a set of six countries. To obtain an endogenous measure of uncertainty and to trace its time-varying impacts on economies jointly, we propose a global vector autoregressive model with drifting coefficients. We assume the shocks to the system to feature a factor stochastic volatility in mean structure, with a scalar driving the time-varying variances of the factors interpreted as macroeconomic uncertainty, similar to Crespo Cuaresma *et al.* (2017). This setup disentangles series-specific volatilities from volatility that is common to all series, and inclusion of the factor volatility in the mean of the process allows to compute impulse response functions to an international uncertainty shock.

From an econometric perspective, we provide several contributions. First, a multi-country model related to the GVAR (see Pesaran *et al.*, 2004) is proposed to account for time-varying static and dynamic interdependencies between economies. Second, we employ Bayesian techniques and adapt global-local priors designed for achieving shrinkage in time-varying parameter models (Belmonte *et al.*, 2014; Bitto and Frühwirth-Schnatter, 2019). Extensions relate to parametric restrictions common in the panel data literature, with a specific focus on extracting cross-sectional information besides overall shrinkage towards sparsity for reliable and precise inference. The setup centers the model on a constant parameter specification with homoscedastic errors and cross-country homogeneity, but allows for data-driven idiosyncrasies along several dimensions. Finally, the high-dimensional variance covariance matrix of the system is modeled using a factor stochastic volatility in mean structure, and the model can thus be considered a multivariate extension of the stochastic volatility in mean model with time-varying parameters in Chan (2017).

Our measure of uncertainty is comparable to established proxies, and correctly identifies known events associated with elevated levels of uncertainty. Considering the idiosyncratic volatilities of country-specific series, we find that the factor stochastic volatility structure discriminates well between events confined to individual economies and overall macroeconomic uncertainty. Moreover, the model detects a substantial degree of homogeneity in macroeconomic dynamics along the cross-sectional dimension. Key insights from the structural analysis of uncertainty shocks are that the responses for prices, unemployment, industrial production and equity prices are heterogeneous across the six countries. We find that uncertainty shocks cause downward pressure on inflation, increase unemployment levels, decrease industrial production and depress equity prices, with differences in timing and magnitude of the effects over the cross section. The terms structure of interest rates generally exhibits decreases in the levels of government bond rates at all maturities, with an accompanying overall flattening of the yield curve. In line with Mumtaz and Theodoridis (2018), the consequences of uncertainty shocks appear to decline gradually for some macroeconomic and financial quantities, while other variables show only little variation in responses over time. We find limited evidence for abrupt changes in the transmission channels of uncertainty shocks.

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A. POSTERIOR DISTRIBUTIONS AND MCMC ALGORITHM

Conditional on the full history of the factors $\{\boldsymbol{f}_t\}_{t=1}^T$ and the loadings $\boldsymbol{L}$, the full system of equations reduces to K unrelated regression models with heteroscedastic errors. This allows for estimation of the system on an equation-by-equation basis, greatly reducing the computational burden compared to full system estimation. To see this, we define $\tilde{\boldsymbol{y}}_t = \boldsymbol{y}_t - \boldsymbol{L}\boldsymbol{f}_t$ and refer to the jth variable of country i by $\tilde{y}_{ij,t}$, which yields

$$\tilde{y}_{ij,t} = \boldsymbol{C}'_{ij,0}\boldsymbol{x}_{it} + \tilde{\boldsymbol{C}}'_{ij,t}\sqrt{\boldsymbol{\Theta}_{ij}}\boldsymbol{x}_{it} + \eta_{ij,t}.$$

Moreover, conditional on the full history of the states $\{\tilde{\boldsymbol{C}}_{ij,t}\}_{t=1}^T$, the innovation variances in $\sqrt{\boldsymbol{\Theta}_{ij}}$ can be treated as standard regression coefficients. For notational simplicity, we define the vector $\boldsymbol{d}_{ij} = (\boldsymbol{C}'_{ij,0}, \sqrt{\theta_{ij,1}}, \dots, \sqrt{\theta_{ij,\tilde{K}}})'$. Let $\bullet$ refer to conditioning on all the other parameters, latent states of the model, and the data; then the posterior distribution of $\boldsymbol{d}_{ij}$ is a multivariate Gaussian,

$$\boldsymbol{d}_{ij}|\bullet \sim \mathcal{N}(\tilde{\boldsymbol{\mu}}_{ij}, \tilde{\boldsymbol{V}}_{ij}). \tag{A.1}$$

The posterior moments are $\tilde{\boldsymbol{V}}_{ij} = (\tilde{\boldsymbol{X}}'_{ij}\tilde{\boldsymbol{X}}_{ij} + \boldsymbol{V}^{-1})^{-1}$ and $\tilde{\boldsymbol{\mu}}_{ij} = \tilde{\boldsymbol{V}}_{ij}(\tilde{\boldsymbol{X}}'_{ij}\tilde{\boldsymbol{Y}}_{ij} + \boldsymbol{V}^{-1}\boldsymbol{\mu})$, with prior moments $\boldsymbol{\mu} = (\mu_{c1}, \dots, \mu_{c\tilde{K}}, \mu_{\theta 1}, \dots, \mu_{\theta\tilde{K}})'$ and $\boldsymbol{V} = \text{diag}(\tau_{c1}, \dots, \tau_{c\tilde{K}}, \tau_{\theta 1}, \dots, \tau_{\theta\tilde{K}})$. The matrix $\tilde{\boldsymbol{X}}_{ij}$ is of dimension $T \times 2\tilde{K}$, with the tth row given by $[\boldsymbol{x}'_{it}, \tilde{\boldsymbol{C}}'_{ij,t} \odot \boldsymbol{x}'_{it}]\exp(-\omega_{ij,t}/2)$, while $\tilde{\boldsymbol{Y}}_{ij}$ is of dimension $T \times 1$ with tth element $\tilde{y}_{ij,t}\exp(-\omega_{ij,t}/2)$. This normalization enables to draw the coefficients from standard posterior quantities for the parameters of homoscedastic linear regression models.

Given draws for the country-specific constant part of the model parameters and the state innovation variances, it is straightforward to obtain the conditional posterior distributions for the prior moments collected in $\boldsymbol{\mu}$ and $\boldsymbol{V}$. Since the results apply to the coefficients in $\boldsymbol{c}_i$ and $\sqrt{\boldsymbol{\theta}}_i$, we again use an indicator $s \in \{c, \theta\}$ and obtain the required quantities for the prior variances

$$\tau_{sj}|\bullet \sim \mathcal{GIG}\left(a_s - N/2, \sum_{i=1}^N (c_{ij} - \mu_{sj})^2, a_s\lambda_s\right), \quad \lambda_s|\bullet \sim \mathcal{G}\left(d_{s0} + k\tilde{K}a_s, d_{s1} + \frac{a_s}{2}\sum_{j=1}^{k\tilde{K}} \tau_{sj}\right),$$

with the local scalings τ_{sj} following a generalized inverse Gaussian distribution and the global shrinkage parameter a Gamma distribution. We proceed with the posterior distribution of the common mean. Conditional on $\{c_{ij}\}_{i=1}^N$, standard methods yield a Gaussian posterior

$$\mu_{sj} \sim \mathcal{N}(\tilde{\mu}_{sj}, \tilde{V}_{sj}),$$

with $\tilde{V}_{sj} = (N\tau_{sj}^{-1} + \tau_{\mu_s j}^{-1})^{-1}$ and $\tilde{\mu}_{sj} = \tilde{V}_{sj}(\sum_{i=1}^N c_{ij}\tau_{sj}^{-1})$. For the prior variance of the common mean, $\tau_{\mu_s j}$, following Griffin and Brown (2010) it is straightforward to obtain

$$\tau_{\mu_s j}|\bullet \sim \mathcal{GIG}\left(a_{\mu_s} - 1/2, \mu_{sj}^2, a_{\mu_s}\lambda_{\mu_s}\right), \quad \lambda_{\mu_s}|\bullet \sim \mathcal{G}\left(d_{\mu_s 0} + k\tilde{K}a_{\mu_s}, d_{\mu_s 1} + \frac{a_{\mu_s}}{2}\sum_{j=1}^{k\tilde{K}} \tau_{\mu_s j}\right).$$

To obtain draws from the posterior distribution of $\sigma_{\omega ij}$ we rely on the methods discussed in Kastner and Frühwirth-Schnatter (2014). Conditional on a realization of $\sigma_{\omega ij}$, the derivation for the required posteriors is similar to the ones above. Specifically, we obtain

$$\tau_{\sigma ij}|\bullet \sim \mathcal{GIG}(a_\sigma - 1/2, \sigma_{\omega ij}, a_\sigma \lambda_\sigma), \quad \lambda_\sigma \sim \mathcal{G}\left(d_{\sigma 0} + K a_\sigma, d_{\sigma 1} + \frac{a_\sigma}{2}\sum_{i=1}^{k}\sum_{j=1}^{N}\tau_{\sigma ij}\right).$$

Note that Eq. (1) conditional on the other parameters of the model is a simple linear regression model with conditionally homoscedastic errors and standard formulae apply (see, for instance, Zellner, 1973). The NG prior employed for the R free elements factor loadings translates to the following posteriors for the corresponding global and local shrinkage parameters:

$$\tau_{Lj}|\bullet \sim \mathcal{GIG}(a_L - 1/2, l_j^2, a_L \lambda_L), \quad \lambda_L \sim \mathcal{G}\left(d_{L0} + R a_L, d_{L1} + \frac{a_L}{2}\sum_{j=1}^{R}\tau_{Lj}\right).$$

We proceed with the posterior distribution for the hyperparameters of the prior on the local scalings $a_\bullet$. Combining likelihood and prior, the conditional posterior for this parameter has no well-known form and we rely on a Metropolis-Hastings step for simulation. By the fact that this step is applicable for the different NG priors on the parameters of the model, for the sake of brevity we refrain from presenting all respective indices and refer again to the various possible index combinations using $\bullet$. Given the support of $a_\bullet$, we propose candidate draws $a_\bullet^*$ from $\mathcal{N}(\ln(a_\bullet), \kappa_\bullet)$, with $\kappa_\bullet$ denoting a tuning parameter that is updated during half of the burn-in period to achieve an acceptance rate between 0.15 and 0.35. Acceptance probabilities are given by

$$\min\left[1, \frac{p(a_\bullet^*)p(a_\bullet^*|\tau_\bullet)a_\bullet^*}{p(a_\bullet)p(a_\bullet|\tau_\bullet)a_\bullet}\right], \tag{A.2}$$

Note that due to the non-symmetric proposal density, the acceptance probability includes a correction term. The respective candidate draw is accepted based on the expression in Eq. (A.2), otherwise the previous draw is retained.

B. MCMC ALGORITHM

Employing the posterior distributions presented in Appendix A, the full MCMC algorithm cycles through the following steps:

1. We simulate the constant part of the VAR coefficients and the process variances of the drifting coefficients jointly equation-by-equation using Eq. (A.1).

2. For the full history of the transformed states $\{\tilde{\boldsymbol{C}}_{ij,t}\}_{t=1}^{T}$, we rely on a forward filtering backward sampling algorithm (see Carter and Kohn, 1994; Frühwirth-Schnatter, 1994). This task that can again be carried out equation-by-equation, conditional on the remaining quantities of the model.

3. Conditional on the country-specific coefficients, it is straightforward to obtain a draw for the common mean $\boldsymbol{\mu}$ and the associated global and local shrinkage parameters to be featured in $\boldsymbol{V}$, employing the distributions presented above. Subsequently, given a simulated value for the common mean, we again draw the global and local shrinkage parameters $\tau_{\mu_s j}$ and λ_s that push the common mean towards sparsity.

4. Simulation of the full history of the idiosyncratic variances $\{\omega_{ij,t}\}_{t=1}^{T}$ is carried out using the algorithm set forth in Kastner and Frühwirth-Schnatter (2014), implemented in the `R`-package `stochvol`. The package moreover draws the innovation variances of the stochastic volatility processes. Conditional on this draw, we use the posterior distribution provided above for obtaining the shrinkage parameters $\tau_{\sigma ij}$ related to the time-varying variances.

5. It is straightforward to simulate from the Gaussian conditional posterior distributions for the factors $\{\boldsymbol{f}_t\}_{t=1}^{T}$. Given the full history of the factors we simulate the free factor loadings in $\boldsymbol{L}$ using standard posteriors. Conditional on a draw of the loadings, we obtain the prior variances τ_{Lj} using the posteriors presented above.

6. The full history for the scalar volatility of the factor $\{h_t\}_{t=1}^{T}$, the proposed measure of uncertainty that also features in the mean of the VAR process, is sampled via an independence Metropolis-Hastings algorithm (Jacquier *et al.*, 2002). A minor adaption required by the notion of the volatility being featured in the mean is accounted for in the respective acceptance probabilities.

7. We update the hyperparameters $a_\bullet$ via Metropolis-Hastings steps sketched above.

For the empirical application, we iterate this algorithm $12,000$ times and discard the initial $6,000$ draws as burn-in. We consider each third draw of the remaining $6,000$, resulting in a set of $2,000$ draws for posterior inference. It is worth mentioning that the algorithm exhibits satisfactory convergence properties.

C. ADDITIONAL RESULTS

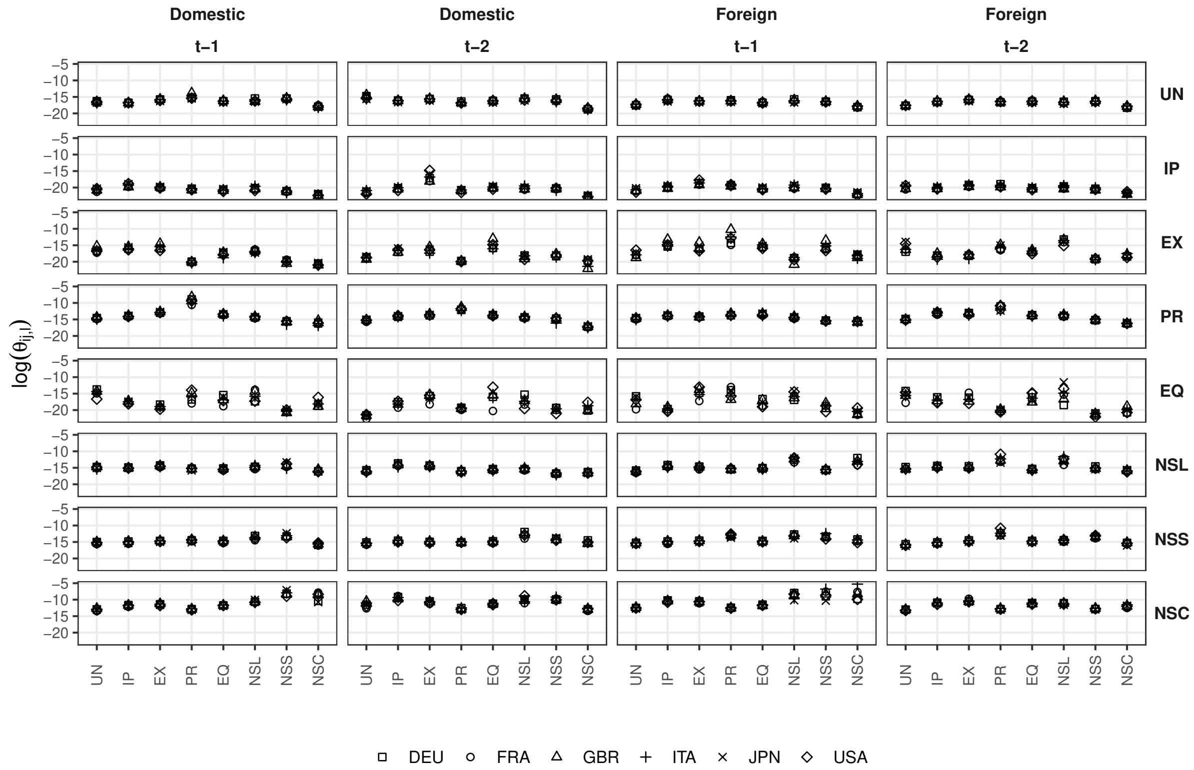

Fig. C.1: Country-specific state innovation variances on the log-scale.

Note: The columns refer to the coefficients associated with a countries' own lagged variables in $\boldsymbol{y}_{it-p}$ (labeled "Domestic") of lag $t-p$, while "Foreign" indicates the coefficients associated with $\boldsymbol{y}^*_{it-q}$ at $t-q$. Countries: Germany (DEU), France (FRA), United Kingdom (GBR), Italy (ITA), Japan (JPN), United States (USA). Variables (rows): Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve.

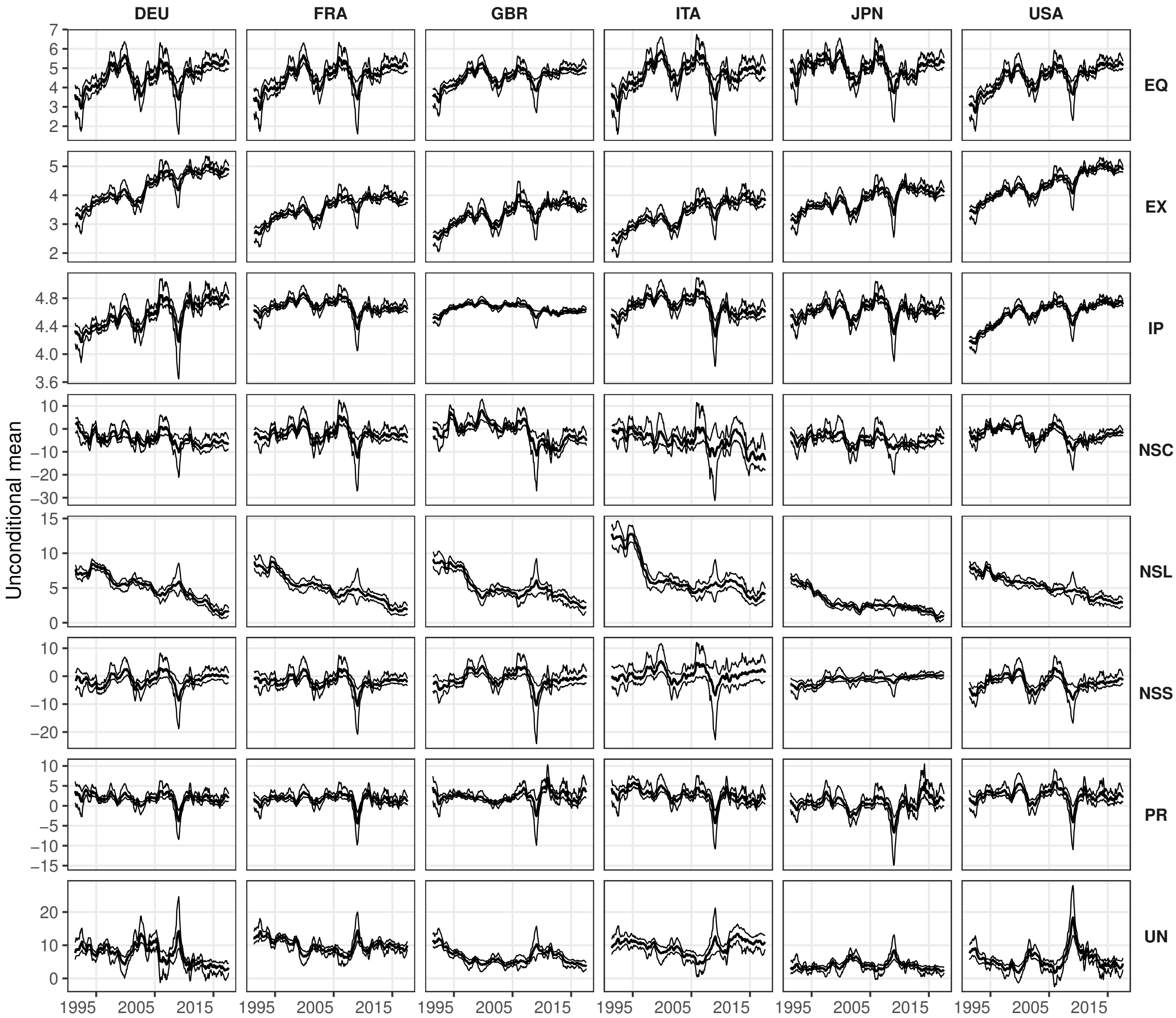

Fig. C.2: Unconditional means over time.

Note: The thick black line depicts the posterior median, alongside the 16th and 84th posterior percentiles (thin lines). Countries (columns): Germany (DEU), France (FRA), United Kingdom (GBR), Italy (ITA), Japan (JPN), United States (USA). Variables (rows): Unemployment (UN), industrial production (IP), exports (EX), consumer price inflation (PR), equity prices (EQ), Nelson-Siegel factors for level (NSL), slope (NSS) and curvature (NSC) of the yield curve.