## MARS – Models, Algorithms, Computers and Systems

Modern high tech research in science and technology requires an interdisciplinary approach. This applies particularly to wide areas of the methodological sciences mathematics and computer science, where typically several aspects of closely related fields of research are considered. These start with a mathematical model, continue with algorithmic problems, and finally cover aspects of the implementation on computers or high performance computing environments and therefore also issues related to the efficiency of computer systems.

MARS is a doctoral programme at the Doctorate School PLUS (DSP Programme), which is organized by the departments of mathematics and computer science of the University of Salzburg. Its objective is to educate doctoral students in the research fields models, algorithms, computers, and systems and also to achieve new insights and research findings especially with regard to the inter-dependency of these fields of research. The focus will be on important topics relevant for the Salzburg research site. MARS fields of research form a cohesive and closely linked line of research and cover a wide spectrum of scientific interests.

Joint activities constitute the structured doctoral program in MARS. These include seminars with external guest speakers, one-day workshops with external guests and multi-day retreats away from the university, as well as summer schools on the topics of MARS.

Doctorate School PLUS
uni-salzburg.at/doktorat

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UNIVERSITÄT SALZBURG

# MARS

**Models, Algorithms, Computers and Systems**

# MARS Workshop

## Adaptive finite elements for optimal control problems

Department of Mathematics
Department of Computer Sciences

University of Salzburg

Fachbereich Mathematik

Fachbereich Computerwissenschaften | UNIVERSITÄT SALZBURG

# Programme and Abstracts

Thursday, January 31st 2019

Seminarraum 2, Naturwissenschaftliche Fakultät

15:00–16:00

## Adaptive optimal control of contact problems

*Andreas Rademacher – TU Dortmund*

In this talk, we present an adaptive method for the optimal control of contact problems. Due to the inherent indifferentiability of contact problems, the need to regularize the problem arises in order to apply fast adjoint-based solution algorithms. By the regularization, an additional error beside the usual finite element discretization and numerical error is introduced. The aim is to construct an adaptive algorithm balancing the different error contributions based on a posteriori error estimators, which are derived using the dual weighted residual (DWR) method. In contrast to the estimation of the discretization and the numerical error, which is quite standard, the estimation of the regularization error is more involved. Using the DWR ansatz, a representation of the regularization error up to a remainder term of higher order arising from the application of the trapezoidal rule is derived, which cannot directly be evaluated. Hence, a numerical approximation based on extrapolation methods using the derivative of the path w.r.t. to regularization parameter is constructed. One advantage of this approach is that the numerical effort corresponds to the one of a single Newton step. In contrast to [Ch. Meyer et al., Adaptive optimal control of the obstacle problem. SIAM Journal of Scientific Computing, 37(2): A918-A945, 2015], we incorporate the estimation of the discretization error and of the numerical one directly in the estimation of the regularization error, see [A. Rademacher and K. Rosin: Adaptive optimal control of Signorini's problem. Computational Optimization and Applications, 70(2): 531-469, 2018] for the details. The main advantage is that the remainder term only depends on the original cost functional and not on the regularized one, which needs not to be bounded. The error estimators are finally utilized in an adaptive algorithm balancing the error contributions. The talk concludes with the application of the presented algorithms to a challenging example and an outlook to the embedding of the presented results into the finite cell framework.

16:00–16:30

## $hp$-FEM for optimal control problems I: Uniform coercivity, unique existence, stability and abstract error estimates

*Andreas Schröder – University of Salzburg*

The first talk introduces a distributed elliptic control problem with control constraints which is formulated as a three field problem and consists of two variational equations for the state and the co-state variables as well as of a variational inequality for the control variable. The adjoint control is associated with the residual of the variational inequality but does not appear in the weak formulation. Each of the three variables may be discretized independently by $hp$-finite elements. In particular, the non-penetration condition of the control variable is relaxed to a finite set of quadrature points. The discretization of the three field problem can be reformulated as a variational inequality for the control variable by using a certain linear operator in $L^2$. In this talk sufficient conditions for the uniform coercivity of this operator are derived in order to show unique existence and stability of the discrete solution. This is, in particular, considered in the context of independent discretizations of the state, co-state and control variables. Furthermore, some abstract error estimates are presented, which enable a priori and a posteriori estimates as introduced in the second talk.

16:30–17:00

## $hp$-FEM for optimal control problems II: A priori and a posteriori error estimates

*Lothar Banz – University of Salzburg*

The second talk extends the results on $hp$-finite elements for optimal control problems with control constraints (as presented in the first talk) by an a priori and an a posteriori error estimate. The proven convergence rates are optimal for the discretization spaces of the state and co-state variables. For the lowest order $h$-version they are also optimal in the control variable. The residual based a posteriori error estimate is reliable and locally efficient even for the uniform $p$-version. Several numerical experiments underline the theoretical results and demonstrate the superiority of $hp$-adaptive finite elements.

17:00–17:30

## $C^0$- and $C^1$-conforming $hp$-finite elements with hanging nodes

*Paolo Di Stolfo – University of Salzburg*

The computation of bases of the standard finite-element space $S$ with reasonably small support is deemed a complicated task if the underlying subdivision admits hanging nodes. In this talk, we give an overview of the multi-level space $M$, which is used in practice as a replacement of $S$. Here, $M$ is defined as the span of a set of continuous, linearly independent functions which can be easily computed on certain subdivisions featuring hanging nodes. We point out some similarities and differences between $S$ and $M$. E.g., the basis functions used for $M$ have a larger support than the typical basis functions used for $S$. Originally, the multi-level construction was introduced to simplify the creation of $C^0$-continuous basis functions. We point out how the same construction can be used to define $C^1$-continuous basis functions on Cartesian meshes, which is the typical setting of the finite cell method.

17:30–18:00

## Discussion

19:00

## Dinner – S'Nockerl Restaurant

Sigmund-Haffner-Gasse 4, Salzburg