Diophantine Triples and Linear Recurrence Sequences

Linear recurrence sequences are used in many fields of research, in theoretical mathematics as well as in applied fields such as physics or computer science. The theory of Diophantine tuples is very old and a purely mathematical one. It has been developed from ancient Greece onwards until now by many number theorists.

My thesis combines these two topics. It contains some interesting new finiteness results about the number of Diophantine triples with values in linear recurrence sequences of a certain shape. It was possible to prove, that if the linear recurrence sequence satisfies the so-called Pisot condition, then in most cases only finitely many Diophantine triples exist. Exceptions can only occur, if the order of the sequence is small and if either the leading coefficient or the leading coefficient times the dominant root is a square in the corresponding number field.

As my main method, I used a version of the Subspace Theorem, which is a well-known and powerful tool in Diophantine number theory. Also some further results from number theory and geometry, like e.g. Bézout's theorem are used in my proofs.

These are the first results on this topic concerning linear recurrence sequences of order larger than 2. They arised as joint works with other number theorists, primarly Dr. Florian Luca and my supervisor Dr. Clemens Fuchs.