Some New Results in Copula Theory

Abstract

This dissertation contains four articles (three of which are already published) in which we tackled a broad range of questions in copula theory and derived various new mathematical results. In the first article (published in the *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*) we studied the old question on sharp inequalities between the two most famous dependence measures: Kendall's τ and Spearman's ρ. The classical inequalities go back to Daniels and Durbin & Stuart - Daniels' inequality was known to be sharp, whereas the inequality by Durbin & Stuart was only known to be sharp on a countable set. Working with so-called shuffles of the minimum copula M we showed that Durbin and Stuart's inequality is not sharp outside this countable set and gave a complete characterisation of the exact τ-ρ-region Ω, the region determined by all possible values of τ and ρ. In particular, we proofed that Ω is not convex and, more surprisingly, for every point $(x, y) \in \Omega$ there exists mutually completely dependent random variables, whose values for Kendall's τ and Spearman's ρ coincide with x and y. In the second article (published in *Extremes*) we studied the ways two-dimensional extreme-value Copulas distribute mass. After investigating the set on which the discrete component concentrates its mass, we showed that the discrete component is non-degenerated if and only if the right-hand derivative D^+A of the Pickands dependence function A has at least one discontinuity point in $(0, 1)$ and proved that it may happen that the discrete component has full support. Finally, we focused on the absolutely continuous and the singular components and showed that the minimum copula M is the only (purely) singular EVC and that there exists a Pickands dependence function A such that the absolutely continuous, the discrete and the singular component have full support. In the third article (published in the *Journal of Multivariate Analysis*) the focus was again on measures of concordance and a potential interrelation with the length profile of copulas. It is well known that Spearman's ρ can be expressed in terms of the area of the lower copula level-sets. We defined the length profile as the map $L_C(t)$ assigning each $t \in [0, 1]$ the length of the respective (lower) level curve and defined ℓ_C as $\int_0^1 L_C(t)\, dt$, i.e. ℓ_C can be interpreted as the average of the length of the level curves. It turned out that ℓ_C is a measure of association, but not a concordance measure in general. Additionally we derived simple formulas for completely dependent copulas and proved that the properties of a measure of concordance hold for some specific classes of copulas. The fourth manuscript (currently under review) focusses on empirical Bernstein copulas and their potential application to the estimation of the dependence measure ζ_1 based on the metric D_1 recently introduced by Trutschnig (2011). The main theorem in this article states that empirical Bernstein copulas can be used to derive a consistent estimator for the dependence measure ζ_1, if the copula C has a Lipschitz continuous first order partial derivative $\frac{\partial C}{\partial u}(u, v)$. A simulation study for different families of copulas showed that the D_1-distance of the empirical Bernstein copula and the copula C itself converges to 0, even if the first order partial derivative is not continuous.