The Generalized Finite Element Method is a discretization scheme that encompasses various approaches for the approximation of solutions to partial differential equations, including higher-order Finite Element Methods, the Extended Finite Element Method and the Fictitious Domain Method. This cumulative thesis deals with a number of methodological contributions to and real world applications of these discretization schemes. Moreover, an algorithm for the efficient generation of domain conforming meshes is proposed for these schemes.

A major difficulty for the application of hp-adaptive Finite Element Methods for meshes based upon quadrilaterals or hexahedrons is an appropriate treatment of (multi-level) hanging nodes as well as unsymmetric refinement patterns, local orientation problems and anisotropic polynomial degrees. The first main contribution of this thesis consists in a concise presentation of the algorithms necessary for an efficient construction of hp-adaptive, H1-conforming finite element spaces based upon Lagrange or integrated Legendre polynomials, to address these difficulties. Numerical results for various benchmark problems indicate that unsymmetric refinements are (as expected) favorable over symmetric refinements to resolve singularities. Optimal refinement ratios seem to depend on the magnitude of these singularities as well as on the chosen (full) tensor product or Serendipity finite element spaces. Furthermore, systems of equations resulting from finite element spaces defined via Lagrange polynomials with Gauss-Lobatto quadrature points as support points yield the lowest measured condition numbers.

The second main contribution of this thesis is given by the definition of a higher-order Extended Finite Element Method. For this purpose, the higher-order Finite Element Method based upon Lagrange as well as integrated Legendre polynomials is discussed in terms of the Partition of Unity Method, a further generalization of the Generalized Finite Element Method. The resulting approach is tested for two benchmark problems as well as an actual real world application using a simple hp-adaptive refinement strategy.

The third and fourth main contribution consist in the development of a higher-order fictitious domain method and a domain conforming mesh generation for the efficient simulation of heat diffusion and thermoelastic deformations in NC milling processes. A major difficulty for this approach is prescribed by the continuously changing domain implied by the material removal that critically depends upon the thermoelastic deformations. Fictitious domain methods seem particularly well suited for this problem setting due to the decoupling of the definition for the (generalized) finite element spaces from meshes for the actual domains via characteristic functions defined on some covering domain. For the efficient evaluation of these characteristic functions in the numerical integration of the fictitious domain method, domain conforming (sub-)meshes are employed. These meshes are obtained from the combination of highly efficient octree techniques with lookup tables for refinement patterns based on possible edge intersection patterns. Numerical simulations compared to a physical NC milling experiment exhibit the applicability of the proposed approach to predict deviations of the milled workpiece from its designed shape due to thermoelastic deformations.