# Abstract


The topic of this thesis is the study of Cauchy-Dirichlet problems of certain non-linear evolutionary partial differential equations. In their most inclusive form, they can be written as

$$\begin{aligned} \partial_t \boldsymbol{\Phi}(u) - \operatorname{div} \mathbf{A}(x,t,u,Du) + \boldsymbol{B}(x,u) &= \operatorname{div} \boldsymbol{F} && \text{in } \Omega_T, \\ u &= u_0 && \text{on } \partial_{\mathrm{par}}\Omega_T. \end{aligned}$$

The equations are considered on a space-time cylinder $\Omega_T := \Omega \times (0,T)$, where $\Omega$ is an open, bounded set in $\mathbb{R}^n$ and $T > 0$ is a positive time. The Cauchy-Dirichlet problem consists of demanding that solutions take prescribed boundary values $u_0$ on the parabolic boundary $\partial_{\mathrm{par}}\Omega_T$, which is composed of the lateral boundary $\partial\Omega \times (0,T)$ as well as the initial boundary $\Omega \times \{0\}$. Inspected subjects are continuity, higher integrability and stability properties.

*Publication I* deals with the equation given by $\boldsymbol{\Phi}(u) = u$, $\mathbf{A}(x,t,u,Du) = D_\xi f(Du)$, $\boldsymbol{B}(x,u) = D_u g(x,u)$, $\boldsymbol{F}(x,t) = 0$ and a time-independent boundary datum $u_0$. We prove a Haar-Rado type theorem, where we conclude that a continuity condition can be transferred from the boundary datum $u_0$ to a variational solution. Additionally, we apply this result to obtain spatial Lipschitz continuity under the assumption that $u_0$ fulfils the bounded slope condition.

In *Publication II*, we inspect the system related to the porous medium equation given by $\boldsymbol{\Phi}(u) = u$, $\boldsymbol{B} = 0$, $\boldsymbol{F} = 0$ and the vector field $\mathbf{A}$ depending on $Du^m$ instead of $Du$. We show that weak solutions are stable under perturbations of the parameter $m$. More precisely, a sequence of weak solutions with parameter $m_i$ will converge to a weak solution with parameter $m$ whenever $m_i \longrightarrow m$ as $i \longrightarrow \infty$. Notably, no higher integrability result is needed to obtain this property.

*Publication III* focuses on weak solutions of the doubly nonlinear problem where $\boldsymbol{\Phi}(u) = |u|^{p-2}u$, $\boldsymbol{B} = 0$ and $u_0$ vanishes when approaching the lateral boundary. Considering the case $p \leq 2$, we show that the weak gradient admits a self-improving property in the sense that it belongs to a better Lebesgue space than what is naturally required. To obtain this higher integrability result, we make use of intrinsically scaled cylinders.