hp-FEM for Elastoplasticity & *hp*-Adaptivity Based on Local Error Reductions

Patrick Bammer

Defensio on 19 February 2024

The first part of the thesis contains the numerical analysis of different hp-finite element discretizations related to two different weak formulations of a model problem in elasto-plasticity with linearly kinematic hardening.
Thereby, the weak formulation either takes the form of a variational inequality of the second kind, including a non-differentiable plasticity functional, or represents a mixed formulation, in which the non-smooth plasticity functional is resolved by a Lagrange multiplier. As the non-differentiability of the plasticity functional causes many difficulties in the numerical analysis and the computation of a discrete solution it seems advantageous to consider discretizations of the mixed formu-lation. The thesis presents an a priori error analysis of a higher-order finite element discretization of the mixed formulation (explicitly including the discretization of the Lagrange multiplier) and the relations between three different hp-discretizations are studied. Furthermore, a reliable a posteriori error estimator that also satisfies some (local) efficiency estimates is derived and an efficient semi-smooth Newton solver is proposed, which is obtained by reformulating a discretization of the mixed formulation as a system of decoupled nonlinear equations.

The second part of the thesis introduces a new hp-adaptive algorithm for solving variational equations associated with elliptic boundary value problems, in which the automatic mesh refinement does not rely on the use of an a posteriori error estimator or smoothness indicators. Instead, the proposed algorithm compares the predicted reduction of the energy error that can be expressed in terms of local modifications of the degrees of freedom in the underlying discrete approximation space. Thereby, the predicted error reduction can be computed by solving computationally inexpensive, low-dimensional linear problems. The concept is presented in an abstract Hilbert space framework, but also applied to hp-finite element discretizations.