PS Algorithms for distributed systems

Exercise Sheet 0

https://avs.cs.sbg.ac.at/

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Exercise 0.1

Let A, B, C be sets. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Exercise 0.2

Let A, B, C, D be sets such that $C \subseteq A$ and $D \subseteq B$. Which of the following statements is true and why? Provide either proof or counterexample.

(a) $(A \setminus C) \times (B \setminus D) \subseteq (A \times B) \setminus (C \times D)$

(b) $(A \setminus C) \times (B \setminus D) \supseteq (A \times B) \setminus (C \times D)$

Exercise 0.3

Let $r \neq 1$ and $n \in \mathbb{Z}_0^+$. Prove by induction that

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

Exercise 0.4

(a) Let $x \in \mathbb{R}$. Prove that $-|x| \leq x \leq |x|$.

(b) Let $a \geq 0$. Prove that $|x| \leq a$ if and only if $-a \leq x \leq a$.

(c) Use the two prior statements to prove the triangle inequality, i.e. $\forall x, y \in \mathbb{R}$ we have $|x + y| \leq |x| + |y|$.

Exercise 0.5

Prove the following statements or provide counterexamples:

(a) $\log(n) = O(n^{o(1)})$.

(b) $n^{(1+\frac{1}{14n})^{7n}} = O(n^{\sqrt{e}})$.

(c) Let $f : \mathbb{R} \to [1, \infty]$ be a function such that $f(n) = \Omega(n)$. Then $\frac{1}{f(n)} = O(n)$.