PS Algorithms for distributed systems

Exercise Sheet 4

https://avs.cs.sbg.ac.at/

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Exercise 4.1

Given an arbitrary network in the asynchronous CONGEST model where a leader has already been elected, prove that we can compute a BFS tree in $O(D^2)$ rounds using $O(m + nD)$ messages.

Exercise 4.2

Let $L \subseteq V$ be a set of leaders. Consider the following algorithm for leader election in a synchronous, anonymous, non-uniform ring with $|V| = n$ nodes:

Algorithm 1 Leader election

```
Set L = V
while |L| > 1 do
    Every node v ∈ L is assigned ID 0 with probability p and ID 1 otherwise
    Every node v ∈ V \ L is assigned ID 2
    Execute Clockwise algorithm with preference for smaller IDs
    Set L to the output of the Clockwise algorithm execution
end while
```

Prove that for a constant $p < 1$ chosen by you, this algorithm can be implemented such that w.h.p. it runs in $O(n \log n)$ rounds using $O(n \log n)$ messages. You can use the leader verification algorithm of exercise 2.2 as a blackbox. In particular, argue why the general bound of $O(n^2)$ is too pessimistic in this case.

Exercise 4.3

A fair coin with $p = \frac{1}{2}$ for heads/tails respectively is tossed $n = 100$ times. If the kth toss yields heads, then $Y_k = 1$ and $Y_k = 0$ otherwise. Consider the number of times the result was heads after n coin tosses: $X_n = Y_1 + Y_2 + ... + Y_n$. Let $\alpha = 0.8$. Give an upper bound for the probability that $X_n \geq \alpha \cdot n = 80$. Try to analyze the probability using each of the following bounds:

1. Markov
2. Tschebyscheff
3. Chernoff

Which method yields the least upper bound?