PS Algorithms for distributed systems

Homework Sheet 2

https://avs.cs.sbg.ac.at/

WS 2024/25

Submit until 13-01-2024 via email.

Homework 2.1

Prove that we can compute a 2-approximation $\hat{D}$ of the unweighted diameter D of a network in the CONGEST model in $O(D)$ rounds, i.e. a number $\hat{D}$ such that $\frac{1}{2}D \leq \hat{D} \leq D$. You may assume a leader has already been determined.

Hint: On an unweighted graph the triangle inequality always holds.

Homework 2.2

Given a graph $G = (V, E)$ with n nodes, prove that the following algorithm computes a $(2k-1)$-spanner $H = (V, F)$ with $O(n^{1+\frac{1}{k}})$ edges:

Algorithm 1 Compute spanner

$F = \emptyset$
while $V \neq \emptyset$ **do**
  Choose any node $s \in V$
  Compute a BFS tree T rooted in s in G
  Compute $L_i(s) = \{v \in V | \operatorname{dist}_G(s, v) = i\}$ for every $i \geq 0$
  Let $i(s)$ be the smallest i such that $|L_i(s)| \leq |L_{i-1}(s)| \cdot n^{\frac{1}{k}}$
  For every node in $L_0(s) \cup ... \cup L_{i(s)}(s)$ add edges to parent in T to F
  Delete all nodes $L_0(s) \cup ... \cup L_{i(s)-1}(s)$ and all incident edges from G
end while

Hint: As a first step, show that $i(s) \leq k$ for any root s of the BFS tree. Note that for $L_i(s) = \emptyset$ the inequality $|L_i(s)| \leq |L_{i-1}(s)| \cdot n^{\frac{1}{k}}$ holds trivially.

Homework 2.3

In the lecture we have seen a randomized algorithm to compute a $(2k-1)$-spanner with $O(n^{1+\frac{1}{k}}+kn)$ edges on an unweighted graph in expected $O(k^2)$ rounds in the CONGEST model by Baswana and Sen. An intermediate result of their algorithm is called a (ρ, μ, ℓ)-Cover where ρ, μ, ℓ are parameters on the stretch and certain edges. Prove that, given an already computed (ρ, μ, ℓ)-Cover, we can compute a $(2\rho-1)$-spanner of G with $O(\mu+\ell n)$ edges in a constant number of rounds if for every cluster every node knows only its parent, children and the ID of its cluster center.

Homework 2.4

Write a program that computes a randomized Maximal Independent Set on an unweighted network in the CONGEST model (i.e. the nodes of the network have a unique ID and the communication via one edge per node per round has a bandwidth limitation of $O(\log(n)$ bits) in $O(D)$ rounds using the Pregel API of the GraphX component of the Apache Spark framework. You can test your code on a self-generated graph created using the graph generator within GraphX. The output shall print the MIS nodes. Submit your code as a single file via email along with your other solutions.