PS Algorithms for distributed systems

Exercise Sheet 7

https://avs.cs.sbg.ac.at/

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Exercise 7.1

Prove that we can compute an $O(\log n)$-approximation of a solution to the APSP problem on an unweighted network in the CONGEST model with n nodes using a Las-Vegas algorithm which runs in $O(n \log n)$ rounds in expectation. This means that if the algorithm terminates, ever node u in the network knows its distance estimate $\delta(u, v)$ such that $\text{dist}(u, v) \leq \delta(u, v) \leq O(\log n) \cdot \text{dist}(u, v)$ for every node $v \in V$. Again, you may assume a leader has already been determined.

Exercise 7.2

Prove that we can compute a $(1 + \epsilon)$-approximation of a solution to the SSSP problem on a weighted network in the CONGEST model with n nodes in $O(\sqrt{nD}\frac{\log^3 n}{\epsilon})$ assuming that the highest weight is polynomial in n, i.e. $W = n^{O(1)}$.

Hint: Simulate an adapted version of Dijkstras algorithm on an overlay network $H = (Z, Z \times Z)$ where the nodes are the centers and the edge weights are the approximate h-distances between the centers.

1 Exercise 7.3

In the lecture we have shown that during the growth phase for a rumor spreading process in the Push model, the growth factor for the number of infected nodes is less than $\frac{7}{6}$ with probability at most $\frac{1}{e^{1/24}}$, i.e. $\Pr[I(t+1) \leq \frac{7}{6}I(t)] \leq \frac{1}{e^{e^{1/24}}}$ under the assumption that $I(t) \leq \frac{n}{3}$. Assuming this inequality, prove that the growth phase lasts $O(\log n)$ rounds.

Hint: Chernoff Bound