PS Algorithms for distributed systems

Homework Sheet 3

https://avs.cs.sbg.ac.at/

WS 2024/25

Submit until 27-01-2025 via e-mail.

Homework 3.1

Given a Las-Vegas algorithm for a problem P for a network with n nodes and diameter D in the CONGEST model with an expected runtime of $R(n)$ rounds, prove that there exists a Monte-Carlo algorithm for P which for every $c \geq 1$ always has a runtime of $O((R(n) + D) \cdot c \log n)$ and outputs a correct solution with high probability.

Hint: Markov Bound

Homework 3.2

Given is a decision problem P and a randomized algorithm $\mathcal{A}$ for P with the following properties:

- For all input $x \in P : \Pr[\mathcal{A}(x) = NO] \leq \frac{1}{3}$
- For all input $x \notin P : \Pr[\mathcal{A}(x) = YES] \leq \frac{1}{3}$

Prove that we can design a Monte-Carlo algorithm which outputs a correct solution to P with high probability.

Hint: Chernoff Bound

Homework 3.3

In the lecture we only analyzed the rumor spreading process for complete graphs. We showed that in both the Push and Pull model we need $\Theta(\log n)$ rounds to infect all n nodes with high probability. Prove that in general for graphs with n nodes, there are unfavorable starting configurations such that the required number of rounds in both the Push and the Pull model can differ by more than a constant factor, using a class of graphs of your choice.

Homework 3.4

Prove that for a rumor spreading process in the Push model on a graph of n nodes, the following is true: If $c' \ln n \leq G(t) \leq \frac{2n}{3}$ for a constant c', then $G(t+1) \leq \frac{9}{10} \cdot G(t)$ holds with high probability.

Hint: The statement is true at least for $c' = 288c$.

Bonus Homework 1

Prove that it is impossible to deterministically compute a Maximal Independent Set in an anonymous ring even if its synchronous and non-uniform.