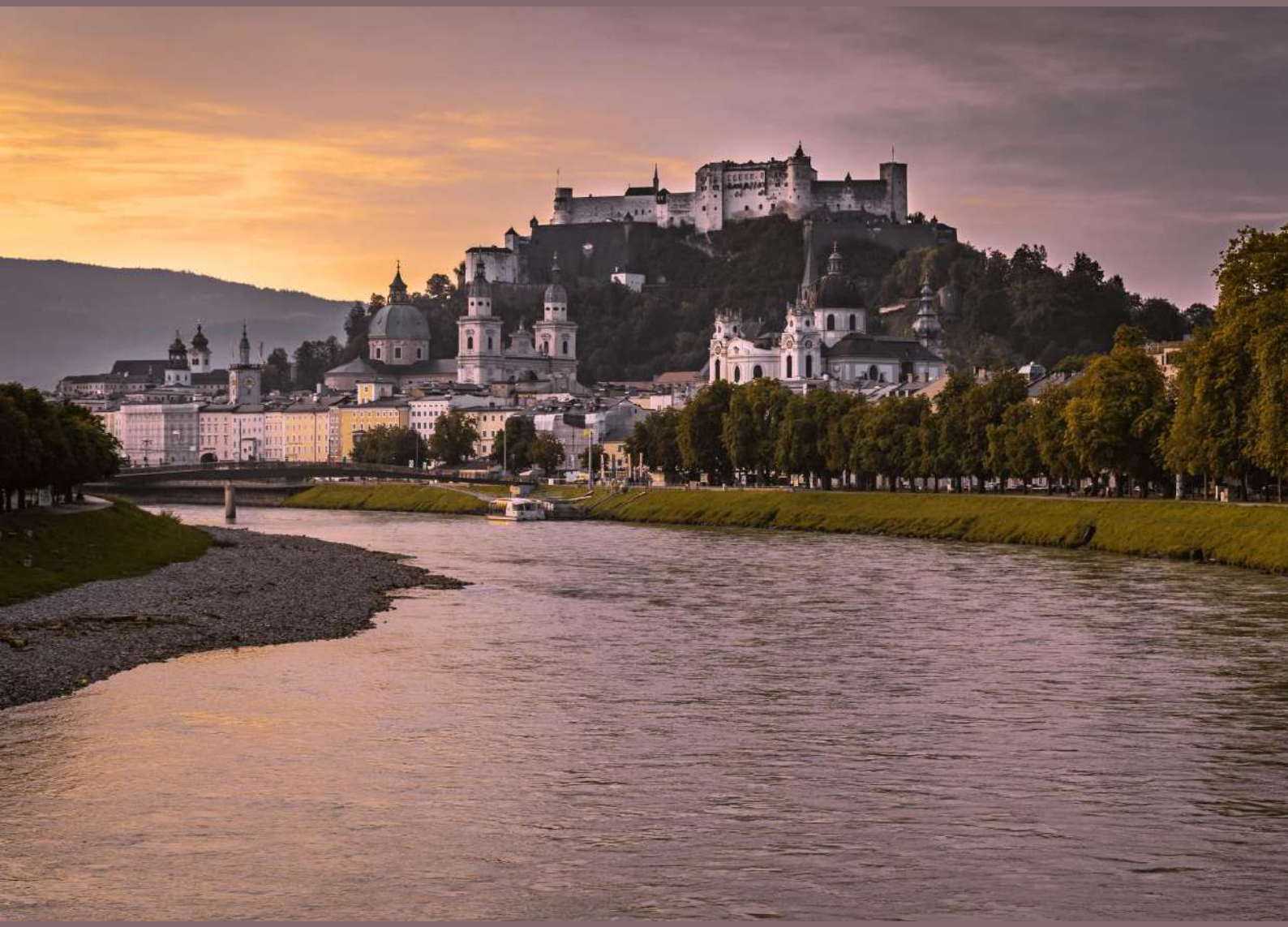
Austrian Numerical Analysis Day 2025

08 - 09 May 2025

Salzburg, Austria



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19th Austrian Numerical Analysis Day

08 - 09 May 2025
Salzburg, Austria

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Location
Unipark Nonntal
Erzabt-Klotz-Straße 1
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Supporting Organization

Department of Mathematics, University of Salzburg

Preface

The 19th Austrian Numerical Analysis Day is being organized by the Department of Mathematics at University of Salzburg and is following the tradition and the spirit of previous events in this series of workshops.

The aim of this workshop is to share current and upcoming research activities in numerical analysis and applied mathematics. Researchers from various Austrian universities and other institutions are invited to present their challenges and discuss their findings. In addition to reinforcing existing collaborations, this annual workshop aims to foster new partnerships.

Lothar Banz and **Andreas Schröder**

Salzburg, May 2025

Contents

Internet Access

If you are from an institution that supports `eduroam` and have your device configured appropriately, you can access the `eduroam` wifi network without any further configuration.

For those who don't have an account yet, an account has been set up to use the **PLUS-Event** wifi network. Once you try to connect **PLUS-Event** wifi network you will be prompted to enter a username and password. The relevant login details are provided.

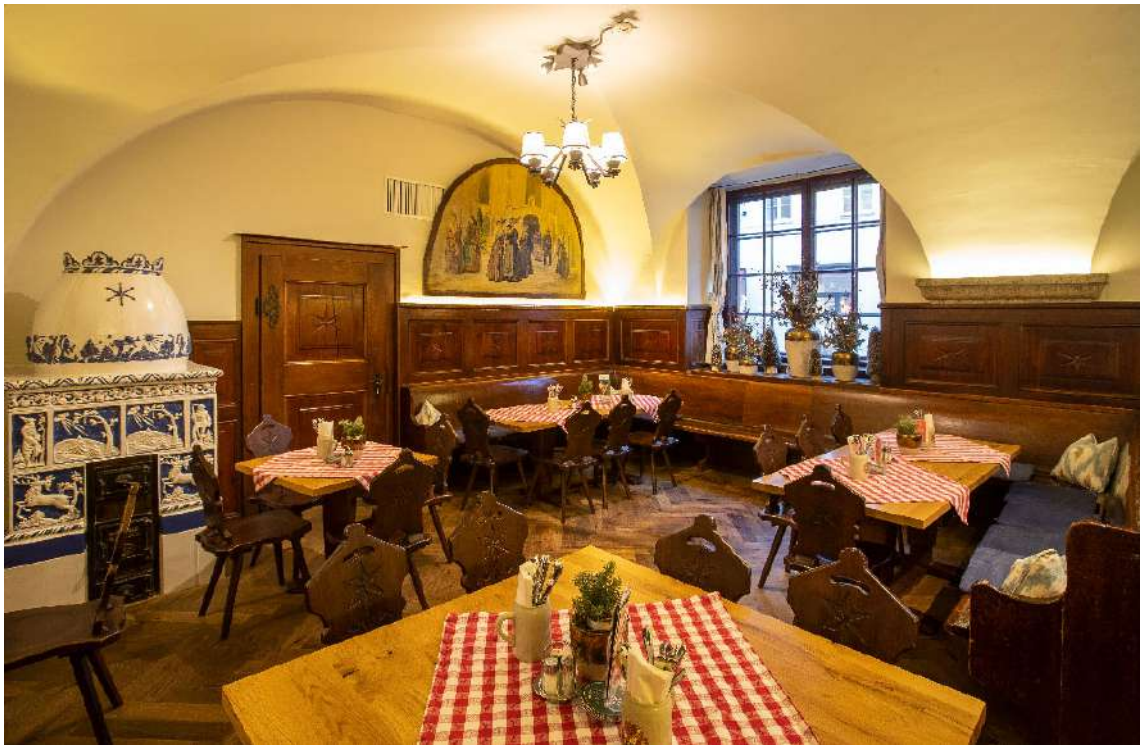
Dinner

Conference Dinner: Sternbräu Restaurant – May 8th, 2025 19:00

Griesgasse 23, 5020 Salzburg.
`https://www.sternbrau.at/`

How to get there from Unipark: The route is about 10-15 minutes by foot, depending on your pace, and takes you through some of the most picturesque parts of Salzburg's Old Town.

There are also several busses that will take you closer to the old town, the best one is bus line 27 (direction Walserfeld) which runs every 15 minutes and will take you from Uni Park to Ferdinand Hanusch Platz (a few minutes' walk from the restaurant).

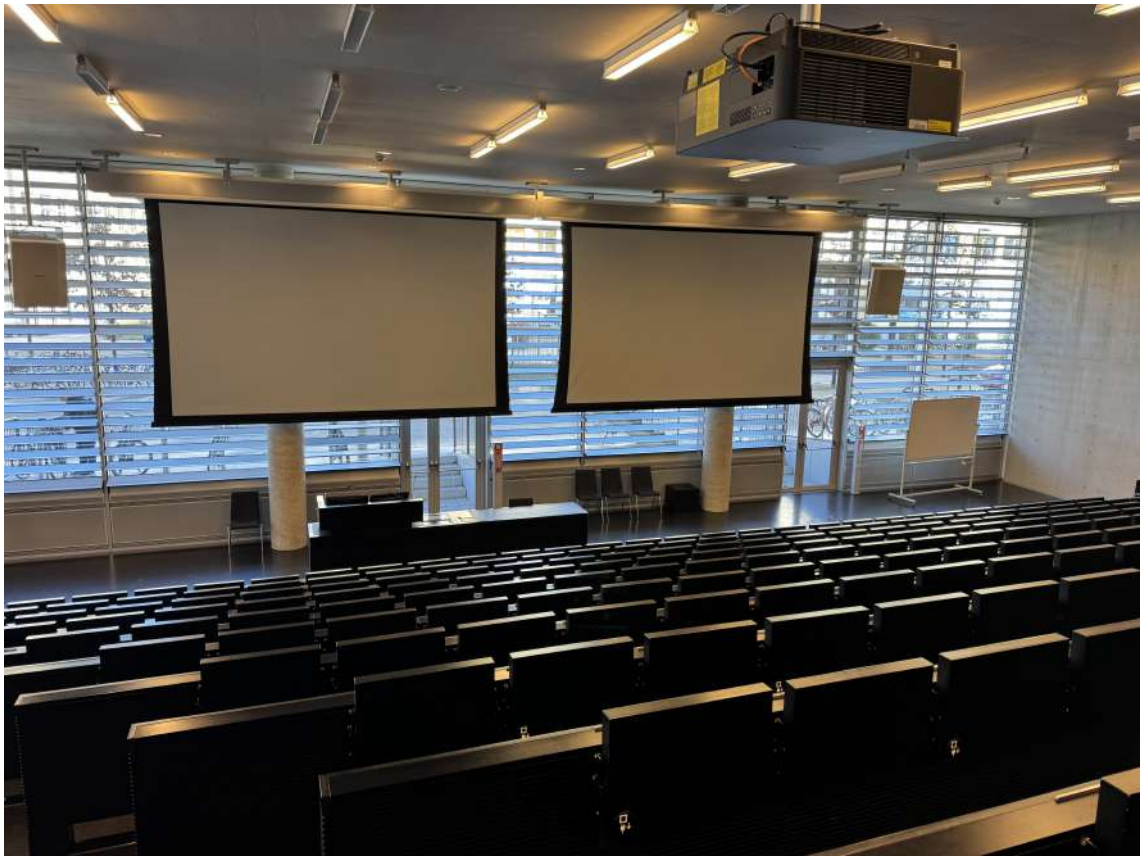


Workshop Location

Unipark Nonntal - Hörsaal Thomas Bernhard

Erzabt-Klotz-Straße 1 – 5020 Salzburg

How to get there from the main train station: Upon arriving at Salzburg Hauptbahnhof, proceed to the bus stops located outside the main station. Board Bus Line 5, which is headed towards Birkensiedlung. After five stops, which will take around 8-10 minutes disembark at the Justizgebäude stop. From the Justizgebäude bus stop, it is a brief walk of about 3-4 minutes to reach Erzabt-Klotz-Straße 1. Alternatively, you may choose to take Bus Line 25 heading towards Untersbergbahn, following the same steps outlined above. The walking distance from the bus stop to the destination will be similar. The conference room Hörsaal Thomas Bernhard is located on the first floor of the Unipark building.



Program

Thursday (May 8th, 2025)

12:45-13:15	*Registration*
13:15	*Opening*
13:20-13:40	**Marvin Fritz** On structure-preserving discretizations of Cahn–Hilliard systems
13:40-14:00	**Syeda Hijab Zahra** The Weighted Essentially Non Oscillatory (WENO) method for numerical simulation of Shallow Water Equations
14:00-14:20	**David Niederkofler** Well-Posedness of Discretizations for Fractional Elasto-Plasticity
14:20-14:40	**Enrico Zampa** Slip and Dirichlet boundary conditions for the Stokes problem in rotational form
14:40-15:00	**Thomas Apel** Discretization of the Stokes Problem with non-homogeneous Dirichlet boundary conditions. Part 1: Weak solutions
15:00-15:20	**Katharina Lorenz** Discretizaion of the Stokes Problem with non-homogeneous Dirichlet boundary conditions. Part 2: Very weak solutions
15:20-16:00	*Coffee Break*
16:00-16:20	**Felix Engertsberger** Reduced basis methods for the simulation of electric machines
16:20-16:40	**Nepomuk Krenn** Multi-physical design optimization of an electric machine
16:40-17:00	**Yu Zhang** Enhancing Efficiency and Stability in Hydrodynamic Simulations with Adaptive Step Size Methods
17:00-17:20	**Fernando Henriquez** Model Order Reduction for Time-Dependent Problems Using the Laplace Transform
17:20-17:40	**Michael Revers** New computational bounds in R.S. Lehman's estimates for the difference $\pi(x) - li(x)$
19:00	*Dinner at Sternbräu*

Friday (May 9th, 2025)

08:20-08:40	**Fatima Hasanova** Multigrid methods for the biharmonic equation on multi-patch domains
08:40-09:00	**Stefan Tyoler** IETI-DP Solvers for Continuous Galerkin Isogeometric Analysis on multi-patch domains with non-matching interfaces
09:00-09:20	**Andreas Schafelner** A parallel-in-time solver for nonlinear degenerate time-periodic parabolic problems
09:20-09:40	**Markus Wess** A Krylov Eigenvalue Solver Based on Filtered Time Domain Solutions
09:40-10:00	**Miriam Schönauer** A Semi-Smooth Newton Solver for an hp-FE Discretization in Elasto-plasticity
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10:40-11:00	**Ani Miraci** Unconditional full linear convergence and optimal complexity of adaptive iteratively linearized FEM for nonlinear PDEs
11:00-11:20	**Alexander Freiszlinger** Convergence of adaptive BEM driven by functional error estimates
11:20-11:40	**Annika Osmers** Goal-oriented adaptive finite cell methods for parabolic problems
11:40-12:00	**Erika Hausenblas** Splitting scheme for stochastic partial differential equations
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14:00-14:20	**Ngoc Tien Tran** A hybrid high-order method for the biharmonic problem
14:20-14:40	**David Wörgötter** Wavenumber-explicit hp-FEM analysis of Maxwell's equations in piecewise smooth media
14:40-15:00	**Paul Stocker** Embedded Trefftz method for the Helmholtz problem
15:00-15:20	**Richard Löscher** Stability analysis of a modified Hilbert transformation for second order initial value problems associated with space-time finite element methods of the wave equation
15:20-15:40	**Pascal Lehner** Well-posedness of a space-time discontinuous Galerkin method for nonlinear acoustics
15:40-16:00	**Olaf Steinbach** Adaptive least-squares space-time finite element methods for convection-diffusion problems
16:00	*Closing*

On structure-preserving discretizations of Cahn–Hilliard systems

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Keywords: *Structure-preserving discretization, Well-posedness, Weak solutions, Cahn–Hilliard system, Nonlocal model, Biot equation, Poroelasticity*

Abstract We present structure-preserving discretizations for two Cahn–Hilliard-type systems: a nonlocal model [2] based on the Ohta–Kawasaki energy and the Cahn–Hilliard–Biot system [1] coupling phase separation with poroelasticity. For the Ohta–Kawasaki equation, we develop a fully discrete scheme preserving essential energy dissipation properties while handling mobility degeneracy through a regularised approximation. The existence of weak solutions is established via limiting arguments, with numerical experiments quantifying how repulsion parameters alter coarsening dynamics in lithography applications. For the Cahn–Hilliard–Biot model we introduce a conforming finite element method that maintains thermodynamic consistency through problem-adapted time splitting. Our scheme decouples the nonlinear Cahn–Hilliard subsystem from linear poroelasticity while preserving mass/volume balances and energy dissipation. Numerical tests demonstrate the preservation of the model's structure in coupled deformation-separation processes. Our methodology provide a template for structure-preserving approximations of coupled phase-field systems with nonlocal interactions and/or multiphysics couplings.

References

[1] A. Brunk and M. Fritz. Structure-preserving approximation of the Cahn–Hilliard–Biot system, *Numerical Methods for Partial Differential Equations*, 41(1):e23159, 2025.

[2] A. Brunk and M. Fritz. Analysis and discretization of the Ohta–Kawasaki equation with forcing and degenerate mobility, *submitted*, arXiv:2411.09498, 2024.

The Weighted Essentially Non Oscillatory (WENO) method for numerical simulation of Shallow Water Equations

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Keywords: *WENO Reconstruction, Non linear Weights, Smoothness Indicator, Shallow Water Equations.*

Abstract

Numerically solving non-linear hyperbolic partial differential equations is challenging due to the presence of discontinuities, which require high-resolution methods [1]. This talk reviews the construction of the WENO scheme, which is designed to solve various PDEs including Shallow Water Equations that model fluid flow in rivers and coastline. The proposed WENO scheme ensures non-oscillatory behavior near discontinuities and maintains $(2r-1)$ order of accuracy in smooth regions. WENO scheme with different non-linear weights, including WENO-JS [2], WENO-Z [3], and WENO-Z + [4] are used to effectively resolve shocks, capture smooth features and maintain numerical stability. High-order point values are computed at the grid interfaces to achieve accurate fluxes. Unlike second-order TVD schemes, which degenerate to first-order accuracy at smooth extrema, WENO schemes maintain high-order accuracy across smooth regions, with first-order convergence at discontinuities. Numerical tests are performed to validate the theoretical results.

References

[1] LeVeque, R. J., Leveque, R. J. (1992). Numerical methods for conservation laws (Vol. 132). Basel: Birkhäuser.

[2] Jiang, G. S., Shu, C. W. (1996). Efficient implementation of weighted ENO schemes. Journal of computational physics, 126(1), 202-228.

[3] Borges, R., Carmona, M., Costa, B., Don, W. S. (2008). An improved weighted essentially non-oscillatory scheme for hyperbolic conservation laws. Journal of computational physics, 227(6), 3191-3211.

[4] Acker, F., Borges, R. D. R., Costa, B. (2016). An improved WENO-Z scheme. Journal of Computational Physics, 313, 726-753.

Well-Posedness of Discretizations for Fractional Elasto-Plasticity

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Keywords: *fractional derivative, elasto-plasticity, well-posedness*

Abstract

We consider a fractional plasticity model based on linear isotropic and kinematic hardening as well as a standard von-Mises yield function, where the flow rule is replaced by a Riesz–Caputo fractional derivative. The resulting mathematical model is typically non-local and non-smooth. Our numerical algorithm is based on the well-known radial return mapping and exploits that the kernel is finitely supported. We propose explicit and implicit discretizations of the model and show the well-posedness of the explicit in time discretization in combination with a standard finite element approach in space. Our numerical results in 2D and 3D illustrate the performance of the algorithm and the influence of the fractional parameter.

Slip and Dirichlet boundary conditions for the Stokes problem in rotational form

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Keywords: *Stokes problem, rotational formulation, slip boundary conditions, Nitsche boundary conditions*

Abstract

The rotational form of Stokes problem, in which the velocity is sought either in $H(\mathrm{curl})$ or $H(\mathrm{div})$, is interesting for many applications, such as semi-Lagrangian methods and enstrophy-conserving schemes. However, it is not clear how to impose slip and Dirichlet boundary conditions in the correct way. In this talk, we show how to reformulate slip boundary conditions as Robin/impedance ones involving the Weingarten map on the boundary of the domain, and we analyze the corresponding continuous and discrete variational formulation. Moreover, we propose a Nitsche type approach to enforce Dirichlet boundary conditions for the $H(\mathrm{curl})$-based problem. In both cases, numerical experiments confirm the predicted convergence rates.

References

[1] W. M. Boon, R. Hiptmair, W. Tonnon and E. Zampa. H(curl)-based approximation of the Stokes problem with slip boundary conditions, *Submitted*, 2024.

[2] W. M. Boon, W. Tonnon and E. Zampa. H(curl)-based approximation of the Stokes problem with Dirichlet boundary conditions. *In preparation.*

Discretization of the Stokes Problem with non-homogeneous Dirichlet boundary conditions. Part 1: Weak solutions

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Keywords: *Stokes equations, non-homogeneous boundary condition, non-convex domain, finite element approximation*

Abstract

This presentation deals with the finite element solution of the Stokes problem

$$\begin{aligned} -\Delta y + \nabla p &= 0 \quad \text{in } \Omega, \\ \nabla \cdot y &= 0 \quad \text{in } \Omega, \\ y &= u \quad \text{on } \Gamma = \partial\Omega. \end{aligned}$$

The polygonal or polyhedral domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, may be non-convex. The aim is to discuss the variants of approximation of non-homogeneous Dirichlet data $u \in H^t(\Gamma)^d$ and to give error estimates in the energy norm and the $L^2(\Omega)$-norm. We assume $t \geq \frac{1}{2}$ such that a weak solution $(y, p) \in H^1(\Omega)^d \times L^2_0(\Omega)$ exists. The case $t \leq \frac{1}{2}$ is considered in Part 2 by Katharina Lorenz.

References

[1] Th. Apel, K. Lorenz, and J. Pfefferer. Numerical analysis for weak and very weak solutions of the Stokes problem with non-homogenuous boundary condition, in preparation.

Discretizaion of the Stokes Problem with non-homogeneous Dirichlet boundary conditions. Part 2: Very weak solutions

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Keywords: *Stokes equations, non-homogeneous boundary condition, non-convex domain, finite element approximation, very weak solution*

Abstract

This talk continues the discussion from Part 1 by Thomas Apel. We consider the Stokes equations

$$\begin{aligned} -\Delta y + \nabla p &= 0 \quad \text{in } \Omega, \\ \nabla \cdot y &= 0 \quad \text{in } \Omega, \\ y &= u \quad \text{on } \Gamma \end{aligned}$$

on a polygonal or polyhedral domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$. In this part, we address the case where the Dirichlet boundary data is not sufficiently regular, specifically when $u \in H^t(\Gamma)^d$ with $t \in [-\frac{1}{2}, \frac{1}{2})$, such that a weak solution cannot be expected. We show how to derive a very weak formulation using the method of transposition. Existence, uniqueness and regularity results are presented.

For the finite element discretization, a regularization method is employed, and the boundary datum is treated using an L^2-projection. Error estimates that show the influence of both the maximal interior angle of the domain and the regularity of the datum are obtained. Numerical experiments are provided to validate the theoretical results.

References

[1] Th. Apel, K. Lorenz, and J. Pfefferer. Numerical analysis for weak and very weak solutions of the Stokes problem with non-homogenuous boundary condition, in preparation.

Reduced basis methods for the simulation of electric machines

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Keywords: *reduced basis methods, harmonic mortaring, linear magnetostatics, electric machines*

Abstract We investigate the efficient realization of electric machine simulations in the linear magnetostatics setting, considering different rotation angles and current excitations. The rotor dynamics are modeled using a harmonic mortaring method [1], which weakly couples the stator and rotor domain via a Lagrange multiplier in the space of trigonometric polynomials. By employing a Schur complement, we reduce the problem to a system for the Lagrange multiplier, significantly improving computational efficiency for varying parameters. To further optimize performance, a greedy algorithm then selects the most relevant harmonic modes. Finally the reduced-order model is validated against the high-fidelity simulation and experimental measurements of a permanent magnet synchronous machine [2].

References

[1] Herbert Egger, Mané Harutyunyan, Richard Löscher and Melina Merkel, Sebastian Schöps. On torque computation in electric machine simulation by harmonic mortar methods, *Journal of Mathematics in Industry*, 12(6), 2022.

[2] Pawan Kumar Dhakal, Kourosh Heidarikani, Annette Muetze, Roland Seebacher. CREATOR Case: Permanent Magnet Synchronous Motor Data, 2024

Multi-physical design optimization of an electric machine

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Peter Gangl
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Keywords: *PDE constrained optimization, topological derivative, electric machines*

Abstract

We aim to optimize the design of a permanent magnet synchronous electric machine, i.e. the distribution of iron, permanent magnets and air in the rotor. Our goal is to

- maximize the average torque to improve efficiency,
- constrain magnet volume to fix costs,
- constrain temperature to avoid magnet damaging,
- constrain magnetic flux density to avoid demagnetization,
- constrain Von Mises stress to avoid mechanical failure.

This results in a multi-material design optimization problem constrained by several PDEs. First we solve the time periodic magnetoquasistatic problem, a nonlinear parabolic PDE, on moving domains to compute the torque. Further we extract from the solution of this problems eddy currents, which serve as source term of a stationary heat equation to compute the temperature. Finally we solve the PDE of linearized elasticity to obtain mechanical stresses subject to centrifugal forces. We represent our material configuration by a vector valued level set function [1] and use the topological derivative, a pointwise sensitivity with respect to material changes, to update the design. For the objectives considered here, we modify the framework [2] to compute the topological derivatives.

References

[1] P. Gangl. A multi-material topology optimization algorithm based on the topological derivative, *CMAME*, 366, 113090, 2020.

[2] P. Gangl, K. Sturm. Automated computation of topological derivatives with application to nonlinear elasticity and reaction–diffusion problems, *CMAME*, 398, 115288, 2022.

Enhancing Efficiency and Stability in Hydrodynamic Simulations with Adaptive Step Size Methods

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Keywords: *Magnetohydrodynamics; Strong stability preserving Runge-Kutta methods; Adaptive time stepping*

Abstract

The adaptive step size method dynamically adjusts the step size of a time integrator based on the behavior of the solution, and is widely used in various scientific and engineering applications. In this work, we introduce this method into ANTARES, a tool for astrophysical and more general hydrodynamical simulations, aiming to enhance both the efficiency and accuracy of numerical simulations. To determine how the step size should be adjusted, we employ an error estimator based on the total energy of the solution, which is evaluated using two different numerical methods. This estimator helps decide whether to decrease or increase the step size in the subsequent iterations, ensuring that the solution maintains reliable accuracy while optimizing computational performance. Specifically, (embedded) pairs of strong stability preserving Runge-Kutta methods are used for error estimation. For illustration the method is applied to magnetohydrodynamics and Rayleigh Benard convection.

Model Order Reduction for Time-Dependent Problems Using the Laplace Transform

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Keywords: *Time-dependent Problems, Model Order Reduction; Reduced Basis Method; Laplace Transform*

Abstract

We propose a reduced basis method for solving time-dependent partial differential equations using the Laplace transform. Unlike traditional approaches, we begin by applying the Laplace transform to the evolution problem, which yields a time-independent boundary value problem that depends on the complex Laplace parameter.

In the offline stage, we systematically sample the Laplace parameter and solve the corresponding set of problems using the Finite Element Method (FEM). We then apply Proper Orthogonal Decomposition (POD) to this set of solutions to obtain a reduced basis of significantly lower dimension than that of the original FEM space. This reduced basis is subsequently used to solve the evolution problem via any appropriate time-stepping method.

Numerical experiments validate our theoretical claims and demonstrate the advantages of the proposed method—both in terms of accuracy and computational efficiency—compared to existing approaches.

New computational bounds in R.S. Lehman's estimates for the difference $\pi(x) - li(x)$

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Keywords: *Prime counting function, Skewes numbers, Rieman zeros, Crossovers*

Abstract

We denote by $\pi(x)$ the usual prime counting function and let $li(x)$ the logarithmic integral of x. In 1966, R.S. Lehman came up with a new approach and an effective method for finding an upper bound where it is assured that a sign change occurs for $\pi(x) - li(x)$ for some value x not higher than this given bound. In this talk we provide further improvements on the error terms including an improvement upon Lehman's famous error term S_3 in his original paper. We are now able to eliminate the lower condition for the size-length η completely. For further numerical computations this enables us to establish sharper results on the positions for the sign changes. We illustrate with some numerical computations on the lowest known crossover regions near 10^{316} and we discuss numerically on potential crossover regions below this value.

References

[1] M. Revers. New bounds in R.S. Lehman's estimates for the difference $\pi(x) - li(x)$, *to appear in: J. Number Theory*, 2025.

[2] M. Revers. New bounds in R.S. Lehman's estimates for the difference $\pi(x) - li(x)$, *ArXiv*, 2501.04488v2, 1-43, 2025.

Multigrid methods for the biharmonic equation on multi-patch domains

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Keywords: *isogeometric analysis, multigrid solvers, biharmonic equations, C^1-smooth discretizations, multi-patch parametrizations*

Abstract

In this talk, we present an ongoing investigation into multigrid solvers for biharmonic equations discretized using isogeometric analysis (IGA). Our primary interest lies in handling C^1-smooth multi-patch domains, which are relevant for fourth-order partial differential equations (PDEs) arising in structural modeling of thin plates and shell structures, discretized with multi-patch spline parameterizations.

Motivated by the works [1] and [3], we explore analysis-suitable G^1 multi-patch parametrizations that facilitate C^1-smooth discretizations. Additionally, we outline a multigrid framework inspired by [4], focusing on efficient two-level refinement relations and the structure of smoothing matrices to optimize computational efficiency. We also discuss prospects for extending these techniques to arbitrary multi-patch surfaces, following the ideas in [2].

This research is part of the ongoing project 'Isogeometric multi-patch shells and multigrid solvers'.

References

[1] A. Collin, G. Sangalli, T. Takacs. Analysis-suitable G^1 multi-patch parametrizations for C^1 isogeometric spaces, *Computer Aided Geometric Design*, 47 (2016) 93–113.

[2] A. Farahat et al. Isogeometric analysis with C^1-smooth functions over multi-patch surfaces, *Computer Methods in Applied Mechanics and Engineering*, 403, Part A (2023) 115706.

[3] M. Kapl, G. Sangalli, T. Takacs. Dimension and basis construction for analysis-suitable G^1 two-patch parameterizations, *Computer Aided Geometric Design*, 52–53 (2017) 75–89.

[4] J. Sogn, S. Takacs. Robust multigrid solvers for the biharmonic problem in isogeometric analysis, *Computers & Mathematics with Applications*, 77, Issue 1 (2019) 105–124.

IETI-DP Solvers for Continuous Galerkin Isogeometric Analysis on multi-patch domains with non-matching interfaces

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Keywords: *Isogeometric Analysis, domain decomposition, h-adaptivity*

Abstract

We propose the **Dual-Primal IsogEometric Tearing and Interconnecting (IETI-DP)** method applied to non-matching multi-patch IgA configurations. The IETI-DP method is a domain decomposition method that extends the **Dual-Primal Finite Element Tearing and Interconnecting (FETI-DP)** approach to multi-patch Isogeometric Analysis. Given that the computational domain is already composed of multiple patches, domain decomposition methods are a natural choice. In simple cases, the trace spaces on all interfaces between any 2 neighbouring patches agree. However, one might consider a more general setting in order to work with, e.g., adaptively refined meshes. Retaining a continuous Galerkin setting, it is feasible to assume that the trace spaces are nested, meaning one of the trace spaces is contained in the opposing trace space.

Initial analysis of IETI-DP methods were established for conforming Galerkin discretizations with completely matching interfaces in [2] and subsequently for discontinuous Galerkin (dG) discretizations with sliding interfaces in [3]. The basis for the analysis is provided by the fundamental algebraic framework developed for FETI-DP methods in [1].

In this talk, we present new results on the IETI-DP method applied to a conforming Galerkin discretization with nested trace spaces on the interfaces. We consider a Schur complement formulation of IETI-DP, which is solved using the conjugate gradient method and introduce suitable preconditioners for the Schur complement in order to show appropriate condition number bounds for the preconditioned system, which are robust in the spline degree, the ratio of patch diameter to grid size, and to some extent also in the diffusion parameter.

Finally, we provide numerical results for a series of benchmark problems in which the grid sizes vary and the diffusion parameter jumps across patch interfaces to illustrate the theoretical estimates.

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A parallel-in-time solver for nonlinear degenerate time-periodic parabolic problems

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Keywords: *parallel-in-time methods, nonlinear time-periodic problems, mesh-independent convergence.*

Abstract

We consider a class of abstract nonlinear time-periodic evolution problems which arise in electrical engineering and other scientific disciplines. We propose an efficient solver for the systems arising after discretization in time based on a fixed-point iteration. Every step of this iteration amounts to the solution of a discretized time-periodic and time-invariant problem for which efficient parallel-in-time methods are available. We establish global convergence with contraction factors independent of the discretization parameters. Together with an appropriate initialization step, we obtain a highly efficient and reliable solver. We illustrate the applicability and performance of the proposed method by simulations of a power transformer, where we further compare with other solution strategies proposed in the literature.

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A Krylov Eigenvalue Solver Based on Filtered Time Domain Solutions

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Keywords: *resonance problem, eigenvalue problem, Krylov method, filtering of time domain solutions*

Abstract

We present a new method for computing eigenvalues and eigenvectors of a generalized Hermitian, matrix eigenvalue problem. The work is focused on large scale eigenvalue problems, where the application of a direct inverse is out of reach. Instead, an explicit time-domain integrator for the corresponding wave problem is combined with a proper filtering and a Krylov iteration in order to solve for eigenvalues within a given region of interest. We report results of small scale model problems to confirm the reliability of the method, as well as the application to large scale acoustic and electromagnetic resonance problems.

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A Semi-Smooth Newton Solver for an *hp*-FE Discretization in Elastoplasticity

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Keywords: *semi-smooth Newton solver, superlinear convergence, hp-finite elements, elastoplasticity*

Abstract

In this paper, we prove superlinear convergence of a semi-smooth Newton solver for a certain class of non-linear systems of equations. Such systems of equations can arise when, in mixed formulations of variational inequalities of the second kind, the Lagrange multiplier constraints are decoupled by using biorthogonal basis functions in a certain *hp*-finite element discretization. Examples of such variational inequalities include elastoplastic problems with linear kinematic hardening. To prove the order of convergence, we formulate the problem in an abstract framework and exploit Schur complement properties. Furthermore, we discuss an *hp*-finite element method in elastoplasticity, verify the superlinear convergence of the semi-smooth Newton solver and demonstrate in numerical examples its applicability and robustness with respect to h, p and the projection parameters.

Unconditional full linear convergence and optimal complexity of adaptive iteratively linearized FEM for nonlinear PDEs

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Keywords: *adaptive finite element method, optimal convergence rates, cost-optimality, iterative linearization, inexact solver, full linear convergence.*

Abstract

In this talk, we present an adaptive iteratively linearized finite element method (AILFEM) in the context of strongly monotone nonlinear operators in Hilbert spaces. The approach combines adaptive mesh-refinement with an energy-contractive linearization scheme and a norm-contractive algebraic solver. We primarily focus on parameter-less choices like the Kačanov linearization method (also called *Picard iteration* in the literature) combined with an optimal geometric multigrid method (or likewise an optimally preconditioned CG method) for the arising system of linear equations. Crucially, we design the adaptive algorithm based on a novel parameter-free algebraic stopping criterion which exploits the energy structure of the problem. This allows to prove as a first result that the adaptively steered number of algebraic solver steps is uniformly bounded. Next, we establish that the adaptive algorithm guarantees full R-linear convergence, meaning that essentially each algorithmic step (either mesh-refinement, linearization, or algebra iteration) contracts a quasi-error quantity consisting of total error and a-posteriori error estimator. Unlike available results requiring sufficiently small adaptivity parameters to ensure even plain convergence, the main and novel result of this approach consists in proving that the new AILFEM algorithm guarantees full R-linear convergence for arbitrary adaptivity parameters. Thus, unconditional convergence is guaranteed. Moreover, for sufficiently small adaptivity parameters (θ steering the mesh-refinement and λ_{lin} controlling the number of linearization steps), the new adaptive algorithm guarantees optimal complexity, i.e., optimal convergence rates with respect to the overall computational cost and hence time. Numerical experiments confirm the theoretical results.

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Convergence of adaptive BEM driven by functional error estimates

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Keywords: *adaptive BEM, a posteriori error control, functional estimates, convergence analysis*

Abstract

In our talk, we present the idea of functional a posteriori error estimates for the boundary element method (BEM) together with a related adaptive mesh-refinement strategy. Unlike most a posteriori BEM error estimators, the proposed functional error estimators are independent of the discretization of the boundary integral equation and, more importantly, do not control the error in the integral density on the boundary (e.g., $||\phi - \phi_h||_{H^{-1/2}(\partial\Omega)}$ for the weakly-singular integral equation), but rather the error of the potential approximation in the domain (e.g., $||\nabla\widetilde{V}(\phi - \phi_h)||_{L^2(\Omega)}$ with the single-layer potential operator $\widetilde{V}\colon H^{-1/2}(\partial\Omega) \to H^1(\Omega)$), which is of greater physical relevance. The estimators rely on the numerical solution of auxiliary problems on strip domains along the boundary, and are based on the boundary residual resulting from BEM. For Galerkin BEM, we sketch the convergence proof, which relies heavily on a priori-convergence of the solutions of the auxiliary problems. Numerical experiments are presented in order to illustrate the theoretical results.

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Goal-oriented adaptive finite cell methods for parabolic problems

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Keywords: *time-dependent domains, finite cell method, a posteriori error control, dual weighted residual method*

Abstract

Partial differential equations defined on time-dependent domains are frequently encountered in the modelling of various industrial processes, such as drilling, grinding, and other manufacturing operations, where the domain geometry changes over time. These processes often require mathematical models that involve complex geometries, which need to be handled using advanced numerical methods for accurate and reliable simulations.

This presentation explores the numerical solution of a parabolic problem in the context of time-dependent domains. The problem is treated as an optimization problem, as outlined in [1]. To make parabolic models with time-varying geometries numerically feasible, a specialized space-time finite element discretization is employed. In this approach, the evolving time-dependent domain is embedded within a larger domain with a constant shape. This method, known as the fictitious domain approach, is combined with the finite element method to form the finite cell method (FCM), as demonstrated in [3]. The FCM allows for efficient and flexible treatment of complex geometries that change over time, making it particularly well-suited for industrial applications.

A significant focus is placed on a posteriori error control using the dual weighted residual method, ensuring the reliability and precision of the results. This technique provides a systematic way to separate and quantify different error sources introduced by the FCM and the numerical solution algorithms, enabling an accurate assessment of the solution's quality. For time-independent optimisation problems, this has been demonstrated in [2]. By combining these numerical methods, the approach enhances accuracy and robustness of simulations in time-dependent industrial scenarios.

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Splitting scheme for stochastic partial differential equations

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Keywords: *Stochastic Partial differential equations, splitting schemes*

Abstract

In the talk, we introduce the numerical approximation of linear stochastic evolution equations with a potential using operator splitting methods. Specifically, we analyze two widely used splitting schemes: the Lie-Trotter and Strang splittings. Our main result provides a representation of the error in terms of Lie brackets, offering insight into how the splitting schemes interact with the underlying operators. We establish the local error structure and use it to derive global error estimates for two different classes of systems: those where one of the operators generates a unitary group and those where it generates an analytic semigroup.

To demonstrate the applicability of our theoretical findings, we apply our results to two important examples: the stochastic Schrödinger equation with a potential and the stochastic Nernst-Planck equation. In both cases, we derive concrete error estimates and specify the conditions on the potential, initial data, and noise that influence the accuracy of the splitting methods. Our analysis highlights the effectiveness of operator splitting techniques in the numerical solution of stochastic partial differential equations.

It is a joint work with Fahim Kistosil.

Automatic partitioning for the low-rank integration of stochastic Boolean reaction networks

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Keywords: *dynamical low-rank approximation, reaction networks, high-dimensional problems, tree tensor networks*

Abstract

The stochastic description of chemical reaction networks with a master equation is important for studying processes in biological cells, but it suffers from the curse of dimensionality: The amount of data stored grows exponentially with the number of chemical species and thus exceeds the capacity of common computational devices for realistic problems. Therefore, time-dependent model order reduction techniques such as the dynamical low-rank (DLR) approximation are needed to tame the complexity.

For the DLR approximation, the reaction network has to be separated into smaller partitions, and each partition is described by a set of low-dimensional basis functions. This approach treats all reaction pathways inside one partition exactly. An approximation is only performed if a reaction crosses the partition boundary. This approximation error is controlled by the so-called *rank*, i.e., the total number of basis functions for each partition. Ideally, the partitions are chosen such that the rank and thus the memory requirements of our approximation are as small as possible. So far, choosing partitions of reaction networks was only done manually by human experts. This however precludes the possibility of rapidly trying out different models as is needed for example in model inference problems.

In this talk, we present an automatic partitioning scheme based on two ingredients. First, a pool of partitionings is generated with the Kernighan–Lin algorithm, which is a heuristic graph partitioning algorithm that minimizes the number of reaction pathways which cross partitions. Second, the information entropy of each partitioning in the pool is computed and the one with minimal entropy is chosen as the "best" partitioning. This algorithm is cheap compared to the time integration of the DLR approximation and thus can be easily integrated into the simulation workflow. We test our scheme by partitioning reaction networks on a single level and also in a hierarchical fashion with tree tensor networks. The resulting accuracy of the scheme is superior to both partitionings chosen by human experts and those found by simply minimizing the number of reaction pathways between partitions.

This talk is based upon the preprint on arXiv:2501.04157.

A hybrid high-order method for the biharmonic problem

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Keywords: *hybrid high-order, biharmonic, a priori, a posteriori, lower eigenvalue bound*

Abstract

We propose a new hybrid high-order discretization for the biharmonic and the corresponding eigenvalue problems. The discrete ansatz space includes degrees of freedom in $n-2$ dimensional submanifolds in addition to the typical degrees in the mesh and on the hyperfaces in the literature, e.g., nodal values in 2d and edge values in 3d. This allows for the characteristic commuting property in the hybrid high-order methodology in any space dimension and, consequently, lower eigenvalue bounds of higher order for the eigenvalue problem. The main results are quasi best-approximation estimates as well as reliable and efficient error control. The latter motivates an adaptive mesh refining algorithm that empirically recovers optimal convergence rates for singular solutions.

Wavenumber-explicit *hp*-FEM analysis of Maxwell's equations in piecewise smooth media

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Keywords: *Pollution effect, hp-FEM, Maxwell equations, wavenumber-explicit regularity*

We consider the high-frequency time-harmonic Maxwell equations with impedance boundary conditions on a domain Ω with analytic boundary. We suppose that Ω consists of multiple subdomains and consider permeability and permittivity tensors that are analytic on every subdomain, but may jump across subdomain interfaces. Under these conditions we show that for any wavenumber $k \geq 1$, a Galerkin discretization based on Nédélec-elements of order p on a mesh with mesh-width h is quasi-optimal, provided there holds the k-explicit scale resolution condition a) that kh/p is sufficiently small and b) that $p/\log k$ is bounded from below.

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Embedded Trefftz method for the Helmholtz problem

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Keywords: *discontinuous Galerkin method, Helmholtz problem, Trefftz method*

Abstract Trefftz discontinuous Galerkin methods provide an efficient approach to discretizing partial differential equations (PDEs) by utilizing basis functions that satisfy the governing equation locally. The Embedded Trefftz method avoids the explicit construction of Trefftz spaces by embedding them within a standard polynomial formulation. This enables the method to handle a broader range of PDEs, including those with non-constant coefficients or inhomogeneous terms, while keeping the implementation simple and ensuring good computational properties.

We present the Embedded Trefftz method for the Helmholtz problem, discussing well-posedness, a priori error estimates, and numerical results.

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Stability analysis of a modified Hilbert transformation for second-order initial value problems associated with space–time finite element methods of the wave equation

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Keywords: *space–time finite element methods, modified Hilbert transformation, wave equation*

Abstract For common discretizations of the wave equation, such as finite element methods in space combined with explicit time-stepping schemes, stability can only be guaranteed if the time step size is chosen sufficiently small relative to the spatial discretization - known as the CFL condition. Unfortunately, this limitation extends to full space-time discretizations, as observed e.g. in [1, 4]. A remedy was proposed in [2], where numerical tests suggest that a suitable transformation of the test space, employing the modified Hilbert transformation, leads to an unconditionally stable scheme. While a partial stability result for the proposed method was previously provided in [3], a comprehensive stability analysis remained an open question.

Building on the analysis from [3], in this talk we aim to present the missing stability analysis for the temporal discretization. Theoretical findings will be complemented by numerical illustrations.

This talk is based on joint work with Matteo Ferrari (Universität Wien) and Marco Zank (TU Wien).

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Well-posedness of a space-time discontinuous Galerkin method for nonlinear acoustics

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Keywords: *nonlinear acoustics, discontinuous Galerkin, discrete well-posedness*

Abstract

In this talk we introduce a space-time discontinuous Galerkin method for a first order in time nonlinear wave equation modeling nonlinear acoustics. The continuous first order in time system is derived in [2], where also its well-posedness is shown in suitable Sobolev spaces.

The discrete system presented here is based on a space-time method for symmetric Friedrichs systems, see [1]. To show its well-posedness we show coercivity of a linearized system and then apply an existence result for Newton's method.

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Adaptive least-squares space-time finite element methods for convection-diffusion problems

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Keywords: *convection-diffusion problems, space-time FEM, least-squares FEM, adaptivity*

Abstract

As in [3] we formulate and analyze a space-time finite element method for the numerical solution of convection-diffusion problems where we consider the convective part as part of the total time derivative. While we can prove stability for conforming space-time finite element spaces, for convection dominant problems we have to use adaptive schemes to resolve the resulting boundary layers. Following [1] we derive a related least-squares formulation, where the discrete adjoint serves as an a posteriori error indicator to drive the adapive refinement scheme. Note that this concept also applies to stationary convection-diffusion problems. Numerical results illustrate the theoretical findings.

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